Securitization and Lending Competition[*]

David M. Frankel (Iowa State University)

Yu Jin (Iowa State University)

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Abstract

We study the effects of securitization on interbank lending competition when banks see private signals of local applicants' repayment chances. If banks cannot securitize, the outcome is efficient: they lend to their most creditworthy local applicants. With securitization, banks lend also to remote applicants with strong observables in order to lessen the lemons problem they face in selling their securities. This reliance on observables is inefficient, raises the mean default risk, and may lead to a deceptive rise in credit scores.

JEL: D82, G14, G21.

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1 Introduction

Securitization of conventional home mortgages began in 1970 with the founding of the Federal Home Loan Mortgage Corporation.[1] The proportion of mortgages held in market-based instruments rose steadily from 20% in 1980 to 68% in 2008.[2] Earlier evidence indicates that securitization has been growing at least since 1975 (Jaffee and Rosen [21, Table 2]).

Remote lending has also grown. Petersen and Rajan [28, Figures I and II] find an upwards trend in distances between small firms and their lenders that began in about 1978 or 1979 and continued through the end of their data in 1992. The mean borrower-lender distance in a sample of small business loans studied by De Young, Glennon, and Nigro [13, pp. 125-6] rose from 5.9 miles in 1984 to 21.5 miles in 2001. Remote lending of residential mortgages also rose from 1992 to 2007 (Loutskina and Strahan [23, p. 1477], discussed below).

We present a tractable theoretical model that links securitization and remote lending. We assume that banks have hard information about all loan applicants but soft information about only local applicants. Without securitization, banks lend only to local applicants because of a winner's curse. With securitization, in contrast, ignorance is bliss: the less a bank knows about its loans, the less of a lemons problem it faces in selling them.[3] This enables banks to compete successfully for some remote applicants.

Our model yields many predictions that are consistent with prior empirical findings (section 5.1):

1. **Securitization Stimulates Lending.** As in Shin [32], securitization leads to

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[1] A detailed history of securitization appears in Hill [20].

[2] The source is unpublished data underlying Figure 3 in Shin [32].

[3] In a prior empirical paper, Loutskina and Strahan [23] point out that banks may have an incentive to lend remotely in order to avoid private information at the time of securitization.

expanded lending by connecting liquid investors with loan applicants. There is considerable evidence that the securitization boom in the 2000s led to expanded lending (Demyanyk and Van Hemert [12]; Krainer and Laderman [22]; Mian and Sufi [24]).

2. **Securitization Favors Remote Lending.** In our model, banks lend remotely only if they can securitize their loans. Moreover, a bank securitizes all of its remote loans but only some of its local loans. Loutskina and Strahan [23] find that as securitization rose, the market share of concentrated lenders - those which originate at least 75% of their mortgages in one MSA - fell from 20% to 4% from 1992 to 2007. Moreover, concentrated lenders retain a higher proportion of their loans. Finally, when they expand to new MSA's, these lenders are more likely to sell their remote loans than those made in their core MSA's.

3. **Remote Borrowers have Strong Observables but High Conditional Default Rates.** While a bank might lend to a local applicant who has a low credit score in our model, it will not do so for a remote one whose credit score is all it sees. Hence, remote borrowers tend to have stronger observables than local borrowers. (We use "borrower" to refer to an applicant who gets a loan.) On the other hand, since banks lack soft information for remote applicants, they make worse lending decisions: conditional on observables, distant borrowers are more likely to default.[4] Loutskina and Strahan [23, p. 1456] find that concentrated lenders (defined above) have lower loan losses despite lending to applicants who are riskier in terms of loan to value ratios. Agarwal and Hauswald [1] find that applicants with strong observables tend to apply online for loans, while in-person applicants tend to be those with weaker observables but positive estimates of the bank's soft information about them.

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[4]This empirical implication is also present in the prior theoretical model of Hauswald and Marquez [18], which we discuss in section 6.3.

Moreover, online loans default more than observationally equivalent in-person loans. De Young, Glennon, and Nigro [13] find that banks that lend remotely have higher default rates.

4. **Securitization Lets Borrowers with Strong Observables Get Cheap Remote Loans.** In our model, securitization encourages banks to lend to remote applicants with strong observables. They must offer low interest rates to these applicants in order to prevent cream skimming by the applicants' local banks. In contrast, banks can demand high interest rates from quality local applicants whose observables are weak since these applicants cannot get remote loans. This has two empirical implications. First, the securitization boom in the 2000s should have strengthened the (negative) relation between borrower observables and interest rates. Rajan, Seru, and Vig [30] find that borrower credit scores and LTV ratios explain just 9% of interest rate variation among loans originated in 1997-2000 but 46% of this variation among loans originated in 2006. A second implication is that remote borrowers pay lower rates.[5] Agarwal and Hauswald [1] find that internet loans carry lower interest rates than in-person loans. Degryse and Ongena [8] find that interest rates decrease with the distance between small firms and their lenders in Belgium. Mistrulli and Casolaro [25] find the same relation among business lines of credit in Italy.

5. **Securitization Raises Conditional and Unconditional Default Rates.** Securitization encourages more remote lending in our model. This raises default rates conditional on borrower observables. Securitization also makes lending more profitable in general, which encourages banks to lower lending standards as in Shin [32]. For both reasons, the unconditional default rate also rises. These predictions are confirmed by empirical research. Rajan, Seru, and Vig [30] find

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[5]The comment in footnote 4 applies here as well.

that conditional default rates rose between 1997-2000 and 2001-6.[6] Demyanyk and Van Hemert [12] find that conditional and unconditional default rates rose from 2001 to 2007.[7]

6. **Securitized Loans Have Higher Conditional Default Rates than Retained Loans.** In our model, local banks adopt lower lending standards in local areas that are more profitable to securitize. Hence, securitized loans have higher default rates than retained loans conditional on observables. Krainer and Laderman [22] find that controlling for observables, privately securitized loans default at a higher rate than retained loans. Elul [15] finds that securitized loans perform worse than observationally similar unsecuritized loans, and that the effect is strongest in the prime market.

In our model, securitization has mixed effects on social welfare. It raises the supply of funding for worthwhile projects by connecting liquid investors with deserving loan applicants. However, it also leads to an inefficient loan allocation by giving banks an incentive to favor remote applicants with strong observables. For instance, consider two applicants in the same location. One has a high credit score but a negative NPV project. The other has a low credit score but a positive NPV project. A remote bank would favor the first applicant since evaluating a project's NPV requires soft information, which it lacks. A local bank may prefer not to fund either applicant because it knows too much about them, which makes their loans difficult to sell. Hence, funds go to the negative-NPV project, which is clearly inefficient.

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[6] They control for the loan interest rate, credit score, loan to value ratio, and dummy variables for adjustable rates, prepayment penalties, and whether the lender lacked documentation of the borrower's income or assets.

[7] Their controls include the loan interest rate, borrower credit score, loan to value ratio, debt to income ratio, local changes in house prices and unemployment since origination, and dummies for prepayment penalties, owner-occupier status, and low documentation.

We treat securitization as an exogenous innovation that encourages remote lending. If instead securitization were initially possible and an exogenous barrier to remote lending were then lifted, our model would also predict a simultaneous increase in both remote lending and securitization.[8] In practice, legal barriers to interstate banking fell gradually starting in Maine in 1978 and ending with the federal government's passage of the Interstate Banking and Branching Efficiency Act of 1994, which abolished all remaining restrictions (Loutskina and Strahan [23, pp. 1451-2]). Since securitization was invented earlier, these barriers may have fallen partly in response to pressure from large banks who were eager to increase their securitization profits. Alternatively, their fall may have been due to an exogenous change in regulatory philosophy. This is an interesting topic for future empirical research.

The rest of the paper is as follows. The model is presented in section 2. Section 3 analyzes a base case without securitization, while the full model is studied in section 4. The model's predictions are discussed and illustrated in section 5. Section 6 reviews related theoretical literature, while conclusions appear in section 7.

2 The Model

A country consist of two *ex ante* identical regions, A and B, each containing a single bank. We will refer to the bank in region A (B) as bank a (respectively, b). Each region $R \in \{A, B\}$ consists of a continuum of locations $\ell \in [0, 1]$. In each location ℓ there is a continuum of agents. All participants are risk-neutral.

Each agent has a project that requires one unit of capital and pays a fixed gross return of $\rho > 1$ if it succeeds and zero otherwise. The project's success probability

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[8]Since we assume banks lack private information about their remote loans and have a lower discount factor than investors, banks securitize all of their remote loans. Since - in our model - they securitize only some of their local loans, removing a barrier to remote lending would raise the proportion of loans that are securitized.

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is the product of the agent's unknown type $\theta \in (0,1)$ and a macroeconomic shock $S_\ell^R \in (0,1)$ to the agent's location ℓ in the region R in which she lives. Project outcomes, conditional on these success probabilities, are independent.[9]

There are four periods, $t = 1, 2, 3, 4$. Period 1 is the lending stage. The banks see signals of each agent's type θ and then make competing loan offers to the agents. This stage determines which agents borrow from which banks, and at what interest rates. Period 2 is the security design stage. Each bank decides which loans to securitize and what liquidating dividend to pay as a function of the returns of these loans. Period 3 is the signalling stage. The bank in each region R first sees signals of its local macroeconomic shocks S_ℓ^R. Each bank then chooses how many shares of its security to sell to investors. Period 4 is the settlement stage: project returns are realized, successful borrowers repay their loans, and each bank pays a liquidating dividend to holders of its security.

The local shock S_ℓ^R has the form

$$S_\ell^R = \sum_{k=1}^{K} \alpha_{k\ell}^R \zeta_k^R. \tag{1}$$

For each k, $\zeta_k^R \in (0,1)$ is a random variable that is realized after the security is sold and $\alpha_{k\ell}^R \in [0,1]$ is a constant satisfying $\sum_{k=1}^{K} \alpha_{k\ell}^R \leq 1$.[10] We refer to ζ_k^R as the kth local factor in region R and to $\alpha_{k\ell}^R$ as location ℓ's loading on this factor. For instance, each factor may represent an industry and the factor loading may be the share of a location's workforce that is employed in the industry.[11] In each region R, the distribution of the factor loading vector $\left(\alpha_{k\ell}^R\right)_{k=1}^{K}$ across locations $\ell \in [0,1]$ has no atoms.[12]

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[9] That is, a project's success probability is θS_ℓ^R regardless of the outcomes of other projects.

[10] One can include a constant term in equation (1) by assuming that one of the factors is a constant.

[11] Factor dependence within and across regions is permitted, as detailed below in section 2.1.3.

[12] That is, there is no factor loading vector that receives a strictly positive probability weight.

At the beginning of period 1, both banks see a public signal $s_{\mathrm{pub}} \in (0, 1)$ of type θ of each agent. Simultaneously, the agent's local bank also sees a private signal $s_{\mathrm{priv}} \in (0, 1)$ of θ.[13] The joint population distribution of the type θ, signals s_{pub} and s_{priv}, and location ℓ is given by a known distribution function F and associated continuous density function f on the domain $(0, 1)^3 \times [0, 1]$.

The assumption that F is region-independent is purely for notational convenience. It could be replaced by region-specific distribution functions F^A and F^B with no change in the results, except for the proliferation of region superscripts throughout the paper. The same is true of all distributions derived from F. In particular, we will also use F to denote the marginal and conditional distribution functions of these variables or subsets of them; for instance, $F(\theta | s_{\mathrm{priv}}, s_{\mathrm{pub}}, \ell)$ denotes the conditional distribution of θ given s_{priv}, s_{pub}, and ℓ. The corresponding densities are written with "f" in place of "F", and we assume that all such densities are continuous.

We assume that an increase in the public signal - or in the private signal *conditional* on the public signal - raises the conditional distribution of θ in a first-order stochastic dominance sense. This is formalized in the following two assumptions. The first says that an increase in the public signal weakly lowers the probability of observing a type θ below any given threshold, and strictly lowers the average of these probabilities across thresholds. Moreover, this effect is bounded above. The second property is like the first but relates to the effect of the private signal on the distribution of types conditional on the public signal. (In both cases, we also condition this distribution on the location ℓ.)

Public Signal Monotonicity For any signal $s_{\mathrm{pub}} \in (0, 1)$ and location $\ell \in [0, 1]$, there are integrable functions $\underline{\lambda} \leq \overline{\lambda} : (0, 1) \to \Re_+$, such that the integral $\int_{\theta=0}^{1} \underline{\lambda}(\theta) \, d\theta$ is strictly positive and for each $\theta \in (0, 1)$, the derivative $\frac{\partial F(\theta | s_{\mathrm{pub}}, \ell)}{\partial s_{\mathrm{pub}}}$ exists and lies between $-\overline{\lambda}(\theta)$ and $-\underline{\lambda}(\theta)$, inclusive.

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[13] The outcome of the model will not depend on what the applicant knows about her own type, as the applicant simply borrows from the bank that offers her the lower interest rate.

Private Signal Monotonicity For any signals $s_{\mathrm{pub}}, s_{\mathrm{priv}} \in (0,1)$ and location $\ell \in$ $[0,1]$, there are integrable functions $\underline{\mu} \leq \overline{\mu} : (0,1) \to \Re_+$ such that the integral $\int_{\theta=0}^{1} \underline{\mu}(\theta) \, d\theta$ is strictly positive and for each $\theta \in (0,1)$, the derivative $\frac{\partial F\left(\theta | s_{\mathrm{priv}}, s_{\mathrm{pub}}, \ell\right)}{\partial s_{\mathrm{priv}}}$ exists and lies between $-\overline{\mu}(\theta)$ and $-\underline{\mu}(\theta)$, inclusive.

Let $\eta = E\left[\theta | s_{\mathrm{pub}}, \ell\right] \overset{d}{=} \eta\left(s_{\mathrm{pub}} | \ell\right)$ denote an agent's expected type given her public signal and location; let $\nu = \eta^{-1} E\left[\theta | s_{\mathrm{pub}}, s_{\mathrm{priv}}, \ell\right] \overset{d}{=} \nu\left(s_{\mathrm{priv}} | s_{\mathrm{pub}}, \ell\right)$ denote the proportional change in this expectation that results from learning her local bank's private signal.[14] By the Law of Iterated Expectations, $E\left(\nu | \eta, \ell\right)$ is identically equal to one.

Henceforth, we will work directly with η and ν, which we refer to respectively as the agent's *credit score* and *private type*. The following result states that (a) the credit score is strictly increasing in the public signal and (b) conditional on the public signal, the private type is strictly increasing in the private signal. Moreover, both rates of increase are bounded.

Claim 1 *The functions $\eta\left(s_{pub} | \ell\right)$ and $\nu\left(s_{priv} | s_{pub}, \ell\right)$ have slopes (with respect to s_{pub} and s_{priv}, respectively) that are strictly positive and finite.*

Claim 1 has the following useful implication. Let us say the pair (η, ℓ) is *feasible* if the location ℓ is in $[0,1]$ and the credit score η lies strictly between $\sup_{s_{\mathrm{pub}}} \eta\left(s_{\mathrm{pub}} | \ell\right)$ and $\inf_{s_{\mathrm{pub}}} \eta\left(s_{\mathrm{pub}} | \ell\right)$. All feasible pairs have a finite, strictly positive probability density:

Claim 2 *The pair (η, ℓ) is distributed according to a finite density g which is strictly positive on the set of feasible pairs (η, ℓ).*

Let the distribution function of (η, ℓ) be denoted $G(\eta, \nu)$. Let the conditional distribution function of the private type ν given the credit score η and location ℓ be

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[14]The symbol "$\overset{d}{=}$" denotes a definition.

denoted $H(\nu|\eta,\ell)$. With probability one, the support of $H(\cdot|\eta,\ell)$ has a finite supre-
mum $\overline{\nu}_{\eta\ell}$.[15] We assume that H is not too concave, and its concavity is nondecreasing
in ν:

No Cream Skimming Let H' and H'' denote the first and second derivatives of
$H(\nu|\eta,\ell)$ with respect to ν. For all feasible pairs (η,ℓ) and for all ν in the
interior of the support of $H(\cdot|\eta,\ell)$, (a) these derivatives exist and (b) $H''\nu/H'$
is greater than -1 and is weakly increasing in ν.

This property will imply that if bank a (for instance) lends to some agents with credit
score η in location ℓ in region B, then bank a prefers to charge an interest rate that
is low enough to deter bank b from lending to any agents in this group. Hence, in
equilibrium bank b does not "cream skim": lend to agents with high private types ν
but not to all agents. This fact allows us to solve analytically for the interest rates
that the banks charge for every credit score, location, and region. It is consistent
with the observation of Agarwal and Hauswald [1] that internet lenders charge low
rates partly in order to prevent cream skimming:

> Arm's-length debt is less readily available but carries lower rates be-
> cause competition among symmetrically informed banks, which rely on
> public information, not only drive down its price but also restrict access
> to credit to minimize adverse selection. [Agarwal and Hauswald [1, p. 2]]

The following result shows that No Cream Skimming is equivalent to a particular
assumption on the primitives of the model.

Claim 3 *Let F' and F'' denote the first and second derivatives of $F(s_{priv}|s_{pub},\ell)$
with respect to s_{priv}. Let ν' and ν'' denote the first and second derivatives of $\nu =$*

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[15] Since $\theta \leq 1$, $\overline{\nu}_{\eta\ell}$ is no greater than $1/\eta$. Since $\theta > 0$, $\eta = E(\theta|s_{\text{pub}},\ell)$ is strictly positive for
any s_{pub} that occurs with positive probability. Hence, $1/\eta$ is finite with probability one, so $\overline{\nu}_{\eta\ell}$ is
as well.

$\nu \left(s_{priv}|s_{pub}, \ell\right)$ *with respect to* s_{priv}. *Assume these derivatives exist. Then No Cream Skimming holds if and only if, for all* s_{priv}, s_{pub}, *and* ℓ, $\frac{F''}{F'\nu'} - \frac{\nu''\nu}{[\nu']^2}$ *is greater than* -1 *and is weakly increasing in* s_{priv}.

The following property states that for any given public signal, one can find private signals that are strong enough that make an agent at least as appealing as any other agent. For instance, if an agent with several loan delinquencies (the public signal) has just inherited a large enough sum of money (the private signal), a bank can ignore her weak credit history.

Limit Irrelevance For any public signal s_{pub}, location ℓ, and $\varepsilon > 0$, there exists a private signal s_{priv} for which $E\left[\theta|s_{\mathrm{pub}}, s_{\mathrm{priv}}, \ell\right] > 1 - \varepsilon$.

This will imply that a remote bank lends to applicants whose credit scores exceed a location-dependent threshold.[16] Indeed, Agarwal and Hauswald [1] find that the chance that a bank will approve an online loan is increasing in both the applicant's public credit quality and the bank's internal assessment, but the latter's effect is very small. Limit Irrelevance permits the depiction of our results using simple two-dimensional diagrams. We also consider what happens in the absence of this assumption.

We now produce an example that satisfies all of the above assumptions. Suppose that s_{priv}, s_{pub}, and ℓ are independent and each is uniformly distributed on the unit interval.[17] This implies that $F\left(s_{\mathrm{priv}}|s_{\mathrm{pub}}, \ell\right) = s_{\mathrm{priv}}$, so $F'' = 0$. Let the conditional distribution of θ given the two signals and location be $F\left(\theta|s_{\mathrm{priv}}, s_{\mathrm{pub}}, \ell\right) = \theta^{\frac{m}{1-m}}$ where $m = 1 - \left(1 - s_{\mathrm{priv}}\right)\left(1 - s_{\mathrm{pub}}\right)$. The mean of this distribution, $E\left(\theta|s_{\mathrm{priv}}, s_{\mathrm{pub}}, \ell\right)$, equals

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[16]Without Limit Irrelevance, a bank may offer loans in a given remote location to applicants with credit score η' but not to those whose credit scores are $\eta'' > \eta'$.

[17]This refers to the closed unit interval in the case of ℓ and the open interval in the case of s_{priv} and s_{pub}.

m. Hence,

$$\nu\left(s_{\mathrm{priv}}\middle|s_{\mathrm{pub}}, \ell\right) = \frac{E\left(\theta\middle|s_{\mathrm{priv}}, s_{\mathrm{pub}}, \ell\right)}{E\left(\theta\middle|s_{\mathrm{pub}}, \ell\right)} = \frac{1 - \left(1 - s_{\mathrm{priv}}\right)\left(1 - s_{\mathrm{pub}}\right)}{1 - \frac{1 - s_{\mathrm{pub}}}{2}},$$

so $\nu'' = 0$ as well. No Cream Skimming then follows from Claim 3. Limit Irrelevance holds since $\lim_{s_{\mathrm{priv}} \to 1} E\left(\theta\middle|s_{\mathrm{priv}}, s_{\mathrm{pub}}, \ell\right) = 1$. Since $\theta^{\frac{m}{m-1}}$ is strictly increasing in m, which is strictly increasing in s_{priv}, Private Signal Monotonicity holds. Public Signal Monotonicity holds since $F\left(\theta\middle|s_{\mathrm{pub}}, \ell\right) = \int_{s_{\mathrm{priv}}=0}^{1} F\left(\theta\middle|s_{\mathrm{priv}}, s_{\mathrm{pub}}, \ell\right) ds_{\mathrm{priv}}$.

2.1 Timing

We now describe each period in greater detail.

2.1.1 Period 1: Lending Stage

In period 1, the banks offer loans first to remote agents and then to local agents. That is, banks a and b first make simultaneous and public loan offers to agents who live in regions B and A, respectively. These offers can depend on an agent's credit score η and location ℓ, which are all the banks know. The banks then make simultaneous and public counter-offers to agents who live in regions A and B, respectively. These offers can depend not only on η and ℓ, but also on an applicant's private type ν and her offer (if any) from her remote bank. Each agent then chooses which, if any, offer to accept. As the banks are perfect substitutes from an agent's point of view, an agent will choose the bank that offers her the lowest gross interest rate as long as it does not exceed the project return ρ.

Let $x_{\eta\ell}^{B}$ equal one if bank a chooses to compete for agents with credit score η in location ℓ in region R and zero otherwise. Let $r_{\eta\ell}^{B}$ be the gross interest rate that bank a offers if $x_{\eta\ell}^{B} = 1$. We assume this rate does not exceed the gross project return ρ, since offering a rate above ρ is equivalent to not making an offer. If the agent did not receive an offer from bank a, then she is willing to pay bank b her gross project return ρ. Thus, with the convention that $r_{\eta\ell}^{B}$ equals ρ whenever bank a does not compete,

$r_{\eta\ell}^B$ equals the willingness to pay of *any* agent. We assume there is an infinitesimal chance that the secondary loan market will be disrupted, forcing the bank to hold all of its loans to maturity. Since only bank b observes an agent's private type ν, this implies that a threshold strategy is optimal: bank b will bid $r_{\eta\ell}^B$ (and win) as long as an agent's private type ν exceeds a threshold $\underline{\nu}_{\eta\ell}^B$ of bank b's choosing. Otherwise, bank b will not bid.

The banks swap roles with respect to agents who live in region A. Let $x_{\eta\ell}^A$ equal one if bank b chooses to compete for agents in region A with credit score η and location ℓ, and zero otherwise. Let $r_{\eta\ell}^A \leq \rho$ equal bank b's bid in period 1 if $x_{\eta\ell} = 1$; set $r_{\eta\ell}^A = \rho$ otherwise. In period 2, bank a responds by choosing thresholds $\underline{\nu}_{\eta\ell}^A$ such that it will lend an agent in region A at interest rate $r_{\eta\ell}^A$ if and only if the agent's private type ν exceeds $\underline{\nu}_{\eta\ell}^A$.

Let C_a^B and X_a^B be the capital cost and realized value, respectively, of bank a's loans to region B:

$$
C_a^B = \int_{\ell=0}^1 \int_{\eta=0}^1 x_{\eta\ell}^B H\left(\underline{\nu}_{\eta\ell}^B | \eta, \ell\right) dG\left(\eta, \ell\right)
$$

$$
X_a^B = \int_{\ell=0}^1 \int_{\eta=0}^1 x_{\eta\ell}^B r_{\eta\ell}^B \left[\eta S_\ell^B \int_{\nu=0}^{\underline{\nu}_{\eta\ell}^B} \nu dH\left(\nu | \eta, \ell\right)\right] dG\left(\eta, \ell\right)
$$

Thus, C_a^B is the integral, over all credit scores η and locations ℓ in region B in which the bank competes (i.e., for which $x_{\eta\ell}^B = 1$), of the measure $H\left(\underline{\nu}_{\eta\ell}^B | \eta, \ell\right)$ of borrowers to whom bank a lends. Likewise, X_a^B is the integral, over all credit scores η and locations ℓ in region B in which bank a competes, of the interest rate $r_{\eta\ell}^B$ charged to these borrowers times their mean probability of repayment (the expression in square brackets).

Likewise, let C_a^A and X_a^A be the capital cost and realized value, respectively, of

bank a's loans to region A:

$$C_a^A = \int_{\ell=0}^1 \int_{\eta=0}^1 \left(1 - H\left(\underline{\nu}_{\eta\ell}^A | \eta, \ell\right)\right) dG\left(\eta, \ell\right)$$

$$X_a^A = \int_{\ell=0}^1 \int_{\eta=0}^1 r_{\eta\ell}^A \left[\eta S_\ell^A \int_{\nu=\underline{\nu}_{\eta\ell}^A}^{\bar{\nu}_{\eta\ell}} \nu dH\left(\nu | \eta, \ell\right)\right] dG\left(\eta, \ell\right)$$

The difference between C_a^A and C_a^B reflects the fact that bank a lends to borrowers in region A whose private types exceed bank a's minimum threshold $\underline{\nu}_{\eta\ell}^A$, while it lends to borrowers in region B if and only if (1) it chooses to compete for them (i.e., only if $x_{\eta\ell}^B = 1$) and (2) their private types are below bank b's minimum threshold $\underline{\nu}_{\eta\ell}^B$. This also explains the difference between X_a^A and X_a^B.

2.1.2 Period 2: Security Design Stage

In period 2, each bank designs one security. The number of shares of each security is normalized to one. We describe this process from the point of view of bank a; bank b's problem is analogous. First, bank a decides what portion of the loans of each identifiable group of borrowers to *securitize*: to include in the pool of assets that underlie its security. Bank a does not know the private types of its borrowers in region B. Hence, for any given credit score η and location ℓ, it must securitize the same proportion of loans to each type $\nu \in \left[0, \underline{\nu}_{\eta\ell}^B\right]$ of borrower in region B. Let this proportion be $p_{\eta\ell}^B$.

As for region A, since a borrower's private type ν is observed by bank a but not by the market, bank a will securitize a loan if and only if the borrower's private type ν is less than some threshold $\bar{\nu}_{\eta\ell}^A$, which must be at least as high as the minimum private type $\underline{\nu}_{\eta\ell}^A$ of borrowers in region A to whom the bank lends. The realized value of bank a's securitized loans is $Y_a = Y_a^A + Y_a^B$ where

$$Y_a^A = \int_{\ell=0}^1 \int_{\eta=0}^1 r_{\eta\ell}^A \left[\eta S_\ell^A \int_{\nu=\underline{\nu}_{\eta\ell}^A}^{\bar{\nu}_{\eta\ell}^A} \nu dH\left(\nu | \eta, \ell\right)\right] dG\left(\eta, \ell\right) \tag{2}$$

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is the realized value of the bank's securitized local loans and

$$Y_a^B = \int_{\ell=0}^{1} \int_{\eta=0}^{1} p_{\eta\ell}^B x_{\eta\ell}^B r_{\eta\ell}^B \left[\eta S_\ell^B \int_{\nu=0}^{\bar{\nu}_{\eta\ell}^B} \nu dH \left(\nu | \eta, \ell \right) \right] dG \left(\eta, \ell \right) \qquad (3)$$

is the realized value of the bank's securitized remote loans. One obtains Y_a^A from X_a^A by replacing the supremum $\bar{\nu}_{\eta\ell}$ of private types ν in X_a^A with the upper bound $\bar{\nu}_{\eta\ell}^A$ on private types ν who are securitized. Similarly, one obtains Y_a^B from X_a^B by multiplying the integrand of the outer double integral in X_a^B by the proportion $p_{\eta\ell}^B$ of loans that are securitized.

After choosing which loans to securitize, each bank $i = a, b$ chooses a function φ_i which determines the ultimate payment per share made by the bank to a holder of its security as a function of the realized loan repayments Y_i of bank i's securitized borrowers. We call $\varphi_i \left(Y_i \right)$ the *payout* of the security. As in DeMarzo and Duffie [10], we assume that φ_i is a nondecreasing function and that both the bank and the market have limited liability: $\varphi_i \left(y \right) \in [0, y]$ for all $y \geq 0$.

There is symmetric information at the security design stage. Why? Let $R \left(i \right)$ denote the region in which bank $i \in \{a, b\}$ is located. While the thresholds $\underline{\nu}_{\eta\ell}^{R(i)}$ and $\bar{\nu}_{\eta\ell}^{R(i)}$ are the private information of bank $i = a, b$, the market can infer the values Y_i^A and Y_i^B of bank i's securitized local and remote loans that result from each pair of factor vectors $\left(\zeta^A, \zeta^B \right)$ in the following way. First, we assume the market observes the measure $1 - H \left(\underline{\nu}_{\eta\ell}^{R(i)} | \eta, \ell \right)$ of bank i's local borrowers for each credit score η and location ℓ, as well as the proportion $\frac{H\left(\bar{\nu}_{\eta\ell}^{R(i)}|\eta,\ell\right) - H\left(\underline{\nu}_{\eta\ell}^{R(i)}|\eta,\ell\right)}{1 - H\left(\underline{\nu}_{\eta\ell}^{R(i)}|\eta,\ell\right)}$ of these borrowers whom bank i securitizes. From these quantities, the market can infer the values $H \left(\underline{\nu}_{\eta\ell}^{R(i)} | \eta, \ell \right)$ and $H \left(\bar{\nu}_{\eta\ell}^{R(i)} | \eta, \ell \right)$ of the distribution function H at the two thresholds. We also assume that for each region R, the market observes the interest rates $r_{\eta\ell}^R$, the lending choices $x_{\eta\ell}^R$, and the remote securitization proportions $p_{\eta\ell}^R$. The market can then use equations (2) and (3), or the corresponding equations for bank b, to compute Y_i^A and Y_i^B for any factor vectors ζ^A and ζ^B.

2.1.3 Period 3: Signalling Stage

In period 3, the banks and investors first see a common public signal $\sigma \in \Re^M$, with unconditional distribution function Ω. Each bank i then sees a private signal $u^i \in \Re_+^N$ of its local factor vector $\zeta^{R(i)} \in (0,1)^K$. The local factor vector $\zeta^{R(i)}$ and the local signal u^i are drawn from a joint density $\gamma\left(\zeta^{R(i)}, u^i | \sigma\right)$, which can depend on the public signal σ as indicated by the notation. However, *conditional* on the public information σ, $\left(\zeta^A, u^a\right)$ and $\left(\zeta^B, u^b\right)$ are independent: the realization of $\left(\zeta^A, u^a\right)$ adds no information about the distribution of $\left(\zeta^B, u^b\right)$ and vice-versa. This is a flexible yet tractable way to permit common or correlated shocks to the two regions.

Let the distribution function of the private signal u^i conditional on the public signal σ be $\Psi\left(u^i | \sigma\right)$. We assume that for all public signals σ, private signals u^i close to the zero vector are observed with strictly positive probability:

$$\inf\left\{u^i \in \Re_+^N : \Psi\left(u^i | \sigma\right) > 0\right\} = 0.$$

Let $\Gamma\left(\zeta^{R(i)} | u^i, \sigma\right)$ be the conditional distribution of the factor vector $\zeta^{R(i)}$ given the private signal u^i and the public signal σ. A higher private signal u^i raises this distribution in the sense of first order stochastic dominance: if $u' \geq u''$, then for all ζ, $\Gamma\left(\zeta | u', \sigma\right) \leq \Gamma\left(\zeta | u'', \sigma\right)$. This implies that for any public signal σ, the worst news bank i can get about its security payout $\varphi_i\left(Y_i\right)$ occurs when its private signal u^i is zero. Finally, we assume that the conditional distribution $\Gamma\left(\zeta^{R(i)} | u^i, \sigma\right)$ is mutually absolutely continuous with respect to the signals (u^i, σ).[18]

The assumption that the density γ and distributions Ψ and Γ are region-independent is for notational convenience. They could be replaced by γ^R, Ψ^R, and Γ^R with no change in the results, except for the proliferation of regional superscripts throughout the paper.

After seeing their signals, the banks choose quantities of their securities to sell.

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[18] This means that the set of realizations of the factor vector $\zeta^{R(i)}$ that can occur with positive probability is independent of the signals u^i and σ.

Bank i's quantity is denoted $q_i \in [0, 1]$. The market (which also sees the public signal σ) uses Bayes's rule to assign a price $p_i = E\left[\varphi_i\left(Y_i\right)|q_a, q_b, \sigma\right]$ to the security of bank $i = a, b$. This is a nonstandard signalling game since the market rationally uses information about bank i's quantity q_i to infer information about bank i's signal u^i, which may be relevant to the value of bank j's security (as it may include some loans to borrowers in bank i's region).

2.1.4 Period 4: Settlement Stage

In period 4, each borrower repays her loan if and only if her project succeeds. These repayments determine the value Y_i of bank i's loan portfolio. Bank i then pays the liquidating dividend $\varphi_i\left(Y_i\right)$ to its investors. While periods 1 through 3 occur at the same point of real time, there is a unit of delay between periods 3 and 4.

2.2 Payoffs

A borrower who pays interest rate r gets $\rho - r$ if her project succeeds and zero otherwise. The banks are liquidity constrained: the discount factor of security buyers, which we normalize to one, exceeds the discount factor of the banks, which is denoted $\delta \in (0, 1)$.[19] The two banks have the same cost of capital, which is normalized to one. In particular, suppose a bank lends c_1 units of capital in period 1 to borrowers who later repay the bank c_4 in period 4. Assume, moreover, that investors pay the bank c_3 in period 3 in return for a security that obligates the bank to pay the investors c_4' in period 4. Then the payoff of investors in the bank's security is $c_4' - c_3$, while the bank's payoff equals $c_3 - c_1 + \delta\left(c_4 - c_4'\right)$: its securitization proceeds c_3, less its capital cost c_1, plus its discounted loan repayments δc_4, less its discounted payment

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[19]This assumption, common in the prior literature, is thought to capture the typical reason cited for why banks sell loans: the availability of attractive alternative investments together with the existence of regulatory capital ratios (e.g., Gorton and Haubrich [16, §III.B]).

$\delta c_4'$ to holders of its security. We assume the investors have at least 2ρ in capital to invest.[20]

2.3 Summary

We now briefly summarize the key features of the model. We focus on region B; analogous choices are made simultaneously in region A with the banks' roles swapped. Consider the group of agents with a given credit score η and location ℓ. In period 1, bank a either offers each such agent a loan at the common interest rate $r_{\eta\ell}^B \in [0, \rho]$ or refrains from competing (whence we set $r_{\eta\ell}^B = \rho$). Bank b then lends, at the interest rate $r_{\eta\ell}^B$, to those agents in the group whose private types exceed a threshold $\underline{\nu}_{\eta\ell}^B$ of bank b's choosing. Agents with lower private types accept a's offer, if any.

In period 2, bank a chooses a proportion $p_{\eta\ell}^B \in [0, 1]$ of its loans to the group to securitize. Bank b securitizes its loans to group members whose private types fall below a threshold $\overline{\nu}_{\eta\ell}^B$ of bank b's choosing Each bank i also specifies a payout function φ_i.

In period 3, each bank $i = a, b$ sees a signal u^i of its local factor vector and then chooses a quantity $q_i \in [0, 1]$ of shares to sell. The market rationally assigns a price p_i to bank i's security using Bayes's rule. In period 4, project returns are realized and successful borrowers repay their loans. Each bank i then pays $\varphi_i(Y_i)$ per share to its security holders, where Y_i equals the repayments of bank i's securitized loans.

3 Base Model: No Securitization

We first analyze a base model without securitization: banks must hold all of their loans to maturity. Bank a's payoff in the base model is simply its discounted loan repayments less its cost of lent capital: $\delta E\left(X_a^A + X_a^B\right) - C_a^A - C_a^B$. Bank b's payoff

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[20]Since each region has a unit measure of loan applicants, each with a project that returns ρ if it succeeds, the securities of the two banks cannot be worth more than 2ρ to the market.

is analogous. In particular, if a bank lends, at a gross interest rate r, to a borrower with credit score η and private type ν living in location ℓ in region $R \in \{A, B\}$, its expected profit is $\delta r \eta \nu E \left(S_\ell^R \right) - 1$: the discounted interest payment δr times the probability $\eta \nu E \left(S_\ell^R \right)$ of project success, less the unitary cost of capital.

In the base model, banks lend only to local agents and extract the full surplus. This is due to the winner's curse: the banks have the same expected payoff from lending to a given agent, but the agent's local bank has superior information about this payoff. Since, by assumption, the local bank makes the second offer, it will slightly underbid the remote bank on profitable loans but refrain from bidding on unprofitable ones. Knowing this, a bank will not make any offers to agents who are not in its region.

Claim 4 *Without the option of securitization, each bank lends only to agents who reside in its own region. Moreover, each borrower's payoff is zero: the gross interest rate on every loan equals the gross project return ρ. An agent gets a loan if and only if her discounted expected gross project return, $\delta \rho \eta \nu E \left(S_\ell^R \right)$, exceeds the bank's unitary cost of capital.*

Without securitization, an agent gets a loan if and only her expected project return exceeds a common threshold. Hence, the allocation of capital to projects is efficient: one agent receives a loan while another does not if and only if the first has a higher expected project return than the second. This efficiency property will not hold with securitization, since a bank may prefer not to lend to a creditworthy agent whom it knows well. Intuitively, the bank's private information about this borrower's repayment probability worsens the lemons problem the bank faces in selling its security.

Our conclusion that all lending is local and the loan allocation is efficient relies on our assumption that the remote bank makes the first offer, followed by the local bank. However, Sharpe [31] obtains the same result with the reverse timing. He assumes

that the remote bank sees not the local bank's offer but rather its offer function: the function from the local bank's signal to its interest rate. If, in addition, the remote bank has no private information about the applicant, then the local bank always posts an offer function that is low enough to make it unprofitable for the remote bank to compete because of a winner's curse (Sharpe [31, Proposition 2, p. 1078]).

4 Full Model

We now turn to the full model, with securitization. We first show that the signalling subgame has a unique separating equilibrium. We then derive formulas for a bank's benefit of securitizing a given loan and of lending to a given borrower when securitization is an option. Finally, we show that any equilibrium of the full model must have a certain intuitive form. We then turn to several computed examples.

4.0.1 The Signalling Subgame

Let $\phi_i\left(u^i, u^j, \sigma\right) = E\left[\varphi_i\left(Y_i\right) | u^i, u^j, \sigma\right]$ be the expected payout of the security of bank $i \in \{a, b\}$, conditional on the signals. ("j" refers to the other bank.) Let $p_i\left(q_i, q_j, \sigma\right)$ be the price offered by the market per unit of bank i's security as a function of the quantities of shares sold by the two banks and the public signal. Bank i's expected securitization profits $\pi^i\left(u^i, q_i, \sigma\right)$, conditional on its signal u^i and quantity q_i and the public signal σ equal the expectation (over all opposing signal vectors u^j) of bank i's gross revenue $q_i p_i\left(q_i, q_j\left(u^j\right), \sigma\right)$ from selling q_i units of the security less its discounted expected payment to the buyers, $\delta q_i \phi_i\left(u^i, u^j, \sigma\right)$:

$$\pi^i\left(u^i, q_i, \sigma\right) = \int_{u^j \in \Re_+^N} \left(q_i\left[p_i\left(q_i, q_j\left(u^j\right), \sigma\right) - \delta \phi_i\left(u^i, u^j, \sigma\right)\right]\right) d\Psi\left(u^j\right).$$

Definition 5 *A Bayes-Nash equilibrium of this game is a pair (q_a, q_b) of measurable quantity functions and a pair (p_a, p_b) of measurable price functions such that:*

1. for $i = a, b$, $q_i(u^i, \sigma) \in \arg\max_q \pi^i(u^i, q, \sigma)$ almost surely;

2. for $i = a, b$, $p_i(q_i(u^i), q_j(u^j), \sigma) = E[\phi_i(u^i, u^j, \sigma) | q_i(u^i), q_j(u^j), \sigma]$ almost surely;

    The equilibrium is separating if, in addition,

3. for $i = a, b$, $p_i(q_i(u^i), q_j(u^j), \sigma) = \phi_i(u^i, u^j, \sigma)$ almost surely.

We restrict to separating equilibria, which satisfy conditions 1 and 3 above. This restriction uniquely determines the banks' behavior and profits. Let $\widehat{\phi}_i(u^i, \sigma) = \int_{u^j \in \Re_+^N} \phi_i(u^i, u^j, \sigma) \, d\Psi(u^j | \sigma)$ and $\pi^i(u^i, \sigma) = \pi^i(u^i, q_i(u^i, \sigma), \sigma)$ be bank i's expected security payout and securitization profits, both conditioned only on bank i's signal u^i and the public signal σ. (In general, $\pi^i(u^i, \sigma)$ may depend on the equilibrium.) The following characterization extends the result of DeMarzo and Duffie [10, eq. (4), p. 79, and Prop. 10, p. 88], which assumes a single bank, to the case of two banks.[21]

Claim 6 *The above double signalling game has a unique separating equilibrium. In it, bank i's expected securitization profits conditional on its signal u^i and the public signal σ are $\pi^i(u^i, \sigma) = (1 - \delta) \widehat{\phi}_i(0, \sigma)^{\frac{1}{1-\delta}} \widehat{\phi}_i(u^i, \sigma)^{-\frac{\delta}{1-\delta}}$. Moreover, each bank i's optimal security design is debt: $\varphi_i(Y_i) = \min\{m_i, Y_i\}$ for some $m_i \in \Re_+$.*

4.0.2 The Benefits of Securitization

Consider either bank $i \in \{a, b\}$. By Claim 6, the realized payout of the bank's security is $\min\{m_i, Y_i\}$ where $Y_i = Y_i^A + Y_i^B$ is the realized value of bank i's securitized loans and m_i is the face value (promised repayment) of the security. Consequently, the expected payout $\widehat{\phi}_i(u^i, \sigma)$ of bank i's security given its signal u^i and the public signal σ is $E[\min\{m_i, Y_i\} | u^i, \sigma]$. Bank i's expected payoff Π_i is the discounted expected return of its loans, less its cost of lending, plus its net securitization profits. By

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[21] It is easy to see that this result generalizes to any finite number of banks.

Claim 6,

$$\Pi_i = \delta E\left(X_i^A + X_i^B\right) - C_i^A - C_i^B + (1-\delta)E\left(\frac{\widehat{\phi}_i(0,\sigma)^{\frac{1}{1-\delta}}}{\widehat{\phi}_i(u^i,\sigma)^{\frac{\delta}{1-\delta}}}\right)$$

In order to understand bank i's incentives to lend to a given agent, one must first consider its benefit from securitizing the agent's loan. To study this, we hold fixed the bank's loan portfolio, and consider the effect of adding a single infinitesimal loan to the bank's security.

Suppose the recipient of this loan has credit score η and private type ν, and lives in location ℓ in region $R \in \{A, B\}$.[22] Let r be the gross interest rate that she must pay if her project succeeds, which occurs with probability $\eta \nu S_\ell^R$. By Claim 6, and the law of iterated expectations, bank i's expected securitization profits are $(1-\delta)E\left[E\left(\widehat{\phi}_i(0,\sigma)^{\frac{1}{1-\delta}}\widehat{\phi}_i(u^i,\sigma)^{-\frac{\delta}{1-\delta}}\middle|\sigma\right)\right]$, where the outer expectation is taken with respect to the public signal σ and the inner conditional expectation is taken with respect to the private signal u^i. The effect, on the bank's profits Π_i, of adding the borrower to the bank's security is thus

$$\Delta\Pi_i = (1-\delta)E\left[E\left(\frac{\widehat{\phi}_i(0,\sigma)^{\frac{1}{1-\delta}}}{\widehat{\phi}_i(u^i,\sigma)^{\frac{\delta}{1-\delta}}}\left(\frac{\Delta\widehat{\phi}_i(0,\sigma)}{\widehat{\phi}_i(0,\sigma)} - \delta\frac{\Delta\widehat{\phi}_i(u^i,\sigma)}{\widehat{\phi}_i(u^i,\sigma)}\right)\middle|\sigma\right)\right]. \quad (4)$$

where for any quantity Q, ΔQ denotes the change in Q that results from adding the loan.

The terms $\Delta\widehat{\phi}_i(0,\sigma)$ and $\Delta\widehat{\phi}_i(u^i,\sigma)$ measure the loan's effect on the expected gross return $\widehat{\phi}_i(u^i,\sigma) = E\left[\min\{m_i, Y_i\}|u^i,\sigma\right]$ of the security in two cases: when the bank's private signal is zero, and when it takes a generic value u^i. In particular, by Claim 6, adding the loan is beneficial insofar as it raises the gross return of the security in the worst case, or lowers it in the generic case. Since higher signals u^i entail higher values of Y_i in a first order stochastic dominance sense, $\widehat{\phi}_i(0,\sigma)$ cannot

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[22] We assume that the market knows the private type ν since it can infer the set of private types that each bank securitizes (p. 15).

exceed $\widehat{\phi}_i(u^i, \sigma)$. Thus, roughly speaking, loans that shrink (raise) the gap between $\widehat{\phi}_i(u^i, \sigma)$ and $\widehat{\phi}_i(0, \sigma)$ must raise (lower) the bank's securitization profits.

The term $\Delta\widehat{\phi}_i(u^i, \sigma) = \Delta E\left[\min\{m_i, Y_i\} \mid u^i, \sigma\right]$ measures the effect of the loan on the bank's expected payment to its security holders, conditional on the signals u^i and σ. This effect occurs entirely through the loan's impact on the realized value Y_i of the bank's securitized loans. First, the security defaults when its face value m_i exceeds the value Y_i of the underlying loans. In this event, the loan raises the security payout by ΔY_i. Second, the loan lowers the chance of default by raising the realized value of the loan portfolio Y_i when this value lies slightly below the face value of the security, m_i. This effect is approximately equal to the product of two terms: the loss $m_i - Y_i$ from default and the probability that Y_i is slightly below m_i. Since both terms are close to zero, this second effect is zero to first order. Hence, the only effect is the first:

$$\Delta\widehat{\phi}_i\left(u^i, \sigma\right) = E\left[1\left(m_i > Y_i\right)\Delta Y_i \mid u^i, \sigma\right], \tag{5}$$

where $1\left(m_i > Y_i\right)$ equals one if $m_i > Y_i$ (if the security defaults) and zero otherwise.

Finally, by (1), the increase in the value Y_i of the underlying assets from adding the borrower is a weighted sum of the macroeconomic factors ζ_k^R that affect region R:

$$\Delta Y_i = r\eta\nu S_\ell^R = r\eta\nu \sum_{k=1}^{K} \alpha_{k\ell}^R \zeta_k^R. \tag{6}$$

Substituting (5) and (6) into (4) and using Claim 6, we find that the effect of securitizing the additional borrower on the bank's payoff is

$$\Delta\Pi_i = r\eta\nu\Omega_{i\ell}^R, \tag{7}$$

where

$$\Omega_{i\ell}^R = E\left(\Omega_{i\ell}^R(\sigma)\right), \tag{8}$$

$$\Omega_{i\ell}^R(\sigma) = E\left[\pi^i\left(u^i, \sigma\right) \mid \sigma\right] \sum_{k=1}^{K} \alpha_{k\ell}^R \left[\Lambda_{ik}^{R0}(\sigma) - \delta\Lambda_{ik}^R(\sigma)\right],$$

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$$\Lambda_{ik}^{R0}\left(\sigma\right)=E\left(\left.\frac{E\left[1\left(m_{i}>Y_{i}\right)\zeta_{k}^{R}\middle|\,u^{i}=0;\sigma\right]}{\widehat{\phi}_{i}\left(0,\sigma\right)}\right|\sigma\right),$$

and

$$\Lambda_{ik}^{R}\left(\sigma\right)=E\left(\left.\frac{\pi^{i}\left(u^{i},\sigma\right)}{E\left(\pi^{i}\left(u^{i},\sigma\right)\middle|\,\sigma\right)}\frac{E\left[1\left(m_{i}>Y_{i}\right)\zeta_{k}^{R}\middle|\,u^{i},\sigma\right]}{\widehat{\phi}_{i}\left(u^{i},\sigma\right)}\right|\sigma\right).$$

By (7), profits from securitizing the loan are the product of four terms. The first is the gross interest rate r: ceteris paribus, it is more profitable to securitize loans that have a higher face value. The second is η: it is more profitable to securitize the loans of borrowers with higher credit scores. The third is ν: borrowers with high private types are also more profitable. The final term is $\Omega_{i\ell}^{R}$ which, by construction, must equal the change in securitization profits from adding a loan for which the product $r\eta\nu$ of the first three terms equals one.

By (8), $\Omega_{i\ell}^{R}$ is the expectation, over all public signals σ, of the change $\Omega_{i\ell}^{R}\left(\sigma\right)$ in securitization profits from adding a loan for which the product $r\eta\nu=1$ and the public signal is σ. $\Omega_{i\ell}^{R}\left(\sigma\right)$, in turn, is the product of the bank's conditional (on the public signal σ) expected securitization profits $E\left[\pi^{i}\left(u^{i},\sigma\right)\middle|\sigma\right]$ and the sum, over all factors k, of the borrower's factor loading $\alpha_{k\ell}^{R}$ times the scaled difference between two terms: $\Lambda_{ik}^{R0}\left(\sigma\right)$ and $\delta\Lambda_{ik}^{R}\left(\sigma\right)$.

The term $\Lambda_{ik}^{R0}\left(\sigma\right)$ is the proportional increase in the lowest conditional expected security payout, $\widehat{\phi}_{i}\left(0,\sigma\right)$, that results from increasing the value Y_{i} of the security's underlying assets by one dollar with probability ζ_{k}^{R}.[23] Thus, $\sum_{k=1}^{K}\alpha_{k\ell}^{R}\Lambda_{ik}^{R0}\left(\sigma\right)$ captures the additional loan's proportional effect on this *worst-case* security payout that is due to the loadings $\alpha_{k\ell}^{R}$ of the borrower's repayment probability on various macroeconomic factors ζ_{k}^{R}. Likewise, $\Lambda_{ik}^{R}\left(\sigma\right)$ is a weighted average over signal vectors u^{i} of the proportional increase in the conditional expected security payout $\widehat{\phi}_{i}\left(u^{i},\sigma\right)$ that results from increasing the value Y_{i} of the security's underlying assets by one dollar with probability ζ_{k}^{R}. Thus, $\sum_{k=1}^{K}\alpha_{k\ell}^{R}\Lambda_{ik}^{R}\left(\sigma\right)$ captures the proportional effect of the

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[23]In the numerator of $\Lambda_{ik}^{R0}\left(s\right)$, the default indicator variable $1\left(m_{i}>Y_{i}\right)$ is present because the additional borrower affects the security value only in the event of default.

additional loan on this *weighted average* security payout that results from the loadings of the borrower's repayment probability on the various macroeconomic factors that affect region R.

By Claim 6, for any public signal σ, the bank's securitization profits are increasing in the expected security payoff conditional on the worst signal vector $u^i = 0$ and decreasing in the expected security payoff for a generic signal vector u^i. For this reason, $\Lambda_{ik}^{R0}(\sigma)$ enters positively in $\Omega_{i\ell}^R(\sigma)$ while $\Lambda_{ik}^R(\sigma)$ enters negatively. The discount factor δ multiplying $\Lambda_{ik}^R(\sigma)$ captures the bank's preference for liquidity: the lower is δ, the stronger are the bank's liquidity needs, and thus the more likely it is that securitizing the additional loan will be worthwhile.

The above results allow us to derive a concise expression for the total expected gross return to bank $i \in \{a, b\}$ from lending to an agent with credit score η and private type ν who lives in location ℓ in region $R \in \{A, B\}$, when securitization is an option. This expected return has two parts. The first is the expected discounted loan repayment by the borrower, $\delta r \eta \nu E\left(S_\ell^R\right)$: the discounted interest rate δr times the probability $\eta \nu E\left(S_\ell^R\right)$ that the loan will be repayed. The second is the value of the bank's option to securitize the loan. By (7), bank i earns an additional $r \eta \nu \Omega_{i\ell}^R$ from securitizing the agent's loan, which it will do if and only if $\Omega_{i\ell}^R > 0$. For any real number c, let c^+ denote the positive part of c: $c^+ = \max\{0, c\}$. The value of the securitization option is $r \eta \nu \left(\Omega_{i\ell}^R\right)^+$, so the bank's gross return from lending to the borrower is

$$r \eta \nu \left[\delta E\left(S_\ell^R\right) + \left(\Omega_{i\ell}^R\right)^+\right]. \tag{9}$$

Bank i knows the private type ν of the borrower only if she lives in the bank's home region. This is a disadvantage of remote lending. However, there is also a potential advantage: the bank does not have private information about remote shocks. Hence, it faces a lemons problem in reselling local loans but not remote loans. In addition, the bank has a preference for liquidity: $\delta < 1$. For the last two

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reasons, it is always profitable to securitize a remote loan:

Claim 7 *Let $i \neq j$ be the two banks. In any equilibrium, it is profitable for bank i to securitize all of its remote loans: $\Omega_{i\ell}^{R(j)} > 0$.*

4.0.3 Main Results

We present results for region B. Identical results hold for region A upon replacing "a" with "b" and vice versa. Let $r_{\eta\ell}^{B*}$ denote the *deterring rate*: the interest rate, offered by bank a, that makes bank b just willing not to lend to the agent with the highest private type (for whom $\nu = \bar{\nu}_{\eta\ell}$) among those with credit score η living in location ℓ in region B. By equation (9), bank b's gross expected return from lending to this borrower at the interest rate r is $r\eta\bar{\nu}_{\eta\ell}\left[\delta E\left(S_\ell^B\right) + \left(\Omega_{b\ell}^B\right)^+\right]$. Setting this equal to the bank's unitary cost of capital and solving for r, we obtain the deterring rate:

$$r_{\eta\ell}^{B*} = (\eta\bar{\nu}_{\eta\ell})^{-1}\left[\delta E\left(S_\ell^B\right) + \left(\Omega_{b\ell}^B\right)^+\right]^{-1} > 0. \tag{10}$$

Now consider the set of borrowers with credit score η in location ℓ in region B. No Cream Skimming implies that if bank a competes for these borrowers, it prefers to lend to all of them: to prevent bank b from skimming the best (highest-ν) borrowers in the group. This requires bank a to bid an interest rate that is no higher than the deterring rate $r_{\eta\ell}^{B*}$. In addition, bank a cannot charge more than the gross project return ρ, which is the most any borrower will pay. On the other hand, any interest rate below the lesser of ρ and $r_{\eta\ell}^{B*}$ permits bank a to capture all of the borrowers in this group. Hence, if bank a competes for these borrowers, it will offer the interest rate $r_{\eta\ell}^B = \min\left\{\rho, r_{\eta\ell}^{B*}\right\}$. By equation (9), Claim 7, and the fact that $E\left(\nu|\eta, \ell\right) = 1$, bank a's profits from lending a unit of capital to this group are

$$\pi_{\eta\ell}^B = \min\left\{\rho, r_{\eta\ell}^{B*}\right\}\left[\delta E\left(S_\ell^B\right) + \Omega_{a\ell}^B\right]\eta - 1. \tag{11}$$

Our first result, which does not assume Limit Irrelevance, is as follows.

Theorem 8 *Consider the group of agents with credit score η in location ℓ in region B.*

1. *Suppose $\pi_{\eta\ell}^B < 0$. In this case,*

    (a) *bank a does not compete for this group;*

    (b) *if bank b's estimate $\nu\eta$ of an agent's type θ exceeds $\left(\rho \left[\delta E \left(S_\ell^B \right) + \left(\Omega_{b\ell}^B \right)^+ \right] \right)^{-1}$, bank b offers her a loan at an interest rate equal to the gross project return ρ, and the agent accepts.[24] Else bank b does not offer the agent a loan. Bank b securitizes all borrowers in this group to whom it lends if $\Omega_{b\ell}^B > 0$ and none of them if $\Omega_{b\ell}^B < 0$.*

2. *Suppose $\pi_{\eta\ell}^B > 0$. In this case,*

    (a) *bank a offers to lend to each agent in the group at the common interest rate $r_{\eta\ell}^B = \min \left\{ \rho, r_{\eta\ell}^{B*} \right\}$;*

    (b) *bank b makes no offers to this group;*

    (c) *all agents in the group accept bank a's offer; and*

    (d) *bank a securitizes all of them.*

Consider the set of borrowers in a given location ℓ in region B. Theorem 8 characterizes the outcome, in the loan market, of borrowers with a *given* credit score η in this set. It does not show how this outcome varies by the credit score η. We now turn to this important question.

The key difficulty is that bank a's profit $\pi_{\eta\ell}^B$ from lending is not necessarily monotonic in the agent's credit score η. This profit is increasing in the deterring rate $r_{\eta\ell}^{B*}$ (equation (11)) which, in turn, is decreasing in the supremum $\bar{\nu}_{\eta\ell}$ of the agent's possible private types ν (equation (10)). However, we have not specified how the

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[24]By definition, $\nu\eta = E\left(\theta | s_{\text{priv}}, s_{\text{pub}}, \ell \right)$ (p. 9).

supremum $\bar{\nu}_{\eta\ell}$ varies with the credit score η. Limit Irrelevance pins this down in a particular way: $\bar{\nu}_{\eta\ell}$ equals the inverse of the credit score η.[25] By (10), the deterring rate, which we now call simply r_ℓ^{B*}, is independent of the credit score η:

$$r_\ell^{B*} = \left[\delta E\left(S_\ell^B\right) + \left(\Omega_{b\ell}^B\right)^+\right]^{-1}. \tag{12}$$

We now present our second result.

Theorem 9 *Assume Limit Irrelevance. Define the threshold*

$$\eta_\ell^B = \frac{1}{\min\left\{\rho, r_\ell^{B*}\right\}\left[\delta E\left(S_\ell^B\right) + \Omega_{a\ell}^B\right]}. \tag{13}$$

1. *If $\eta < \eta_\ell^B$, then $\pi_{\eta\ell}^B < 0$: bank a does not compete for this group. If bank b's estimate $\nu\eta$ of an agent's type θ exceeds $\left(\rho\left[\delta E\left(S_\ell^B\right) + \left(\Omega_{b\ell}^B\right)^+\right]\right)^{-1} = r_\ell^{B*}/\rho$, bank b offers her a loan at an interest rate equal to the gross project return ρ, and the agent accepts. Else bank b does not offer the agent a loan. Bank b securitizes all borrowers in this group to whom it lends if $\Omega_{b\ell}^B > 0$ and none of them if $\Omega_{b\ell}^B < 0$.*

2. *If $\eta > \eta_\ell^B$, then $\pi_{\eta\ell}^B > 0$: bank a offers all borrowers in this group the same interest rate $\min\left\{\rho, r_\ell^{B*}\right\}$. Bank b does not compete and all agents accept bank a's offer. Moreover, bank a securitizes all loans to this group.*

Proof. By Limit Irrelevance, $\bar{\nu}_{\eta\ell} = 1/\eta$, so $r_{\eta\ell}^{B*} = r_\ell^{B*}$. By equations (11) and (13), $\pi_{\eta\ell}^B = \eta/\eta_\ell^B - 1$. Hence, $\pi_{\eta\ell}^B \gtrless 0$ as $\eta \gtrless \eta_\ell^B$. The rest follows from Theorem 8 and equation (12). ∎

Under Limit Irrelevance, bank a lends to an agent in region B if and only if her credit score η exceeds the location-dependent credit threshold η_ℓ^B. This threshold is decreasing in bank a's securitization profits, as captured by $\Omega_{a\ell}^B$, and weakly decreasing in bank b's securitization profits, as captured by $\left(\Omega_{b\ell}^B\right)^+$. If bank a's securitization

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[25] This is because $\bar{\nu}_{\eta\ell} = \eta^{-1} \sup_{s_{\mathrm{priv}}} E\left[\theta|s_{\mathrm{pub}}, s_{\mathrm{priv}}, \ell\right] = 1$.

profits are low relative to those of bank b, it is harder for bank a to compete with bank b. Bank a responds by competing for fewer borrowers in location ℓ: it raises its threshold.[26]

By part 2 of Theorem 9 and equation (12)), bank a offers the interest rate $\min\left\{\rho, \left[\delta E\left(S_\ell^B\right) + \left(\Omega_{b\ell}^B\right)^+\right]^{-1}\right\}$ if an agent's credit score is above a's threshold. This is weakly decreasing in bank b's securitization profits, as captured by $\left(\Omega_{b\ell}^B\right)^+$.[27] Intuitively, if bank b is eager to securitize loans to the given location, then bank a must offer a low interest rate in order to keep bank b out.

A key prediction of Theorem 9 is that a bank will use a credit score threshold in deciding on remote loan applications. This feature survives a considerable weakening of Limit Irrelevance. As long as bank a's profit $\pi_{\eta\ell}^B$ equals zero at a unique value of η, a threshold policy is optimal.[28] By equations (10) and (11), a sufficient condition for this is that $\bar{\nu}_{\eta\ell}$ - the maximum proportional increase in the agent's expected type θ that comes from learning her private signal σ_{priv} - be decreasing in η. This seems plausible; for instance, knowing that a loan applicant comes from a good family would seem to raise her chances of repaying a loan by a smaller proportion if her credit record is already quite strong.

5 Illustrations

We now discuss the implications of Theorem 9 for the effects of securitization, comparative statics, and efficiency under Limit Irrelevance. We illustrate these results in a series of figures. The figures - but not the discussion - rely on the following additional assumptions.

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[26] This occurs, in particular, if location ℓ in region B has low loadings on factors about which bank b will be well informed when it decides how much of its security to sell.

[27] By part 2 of Theorem 9 and equation (12)), bank a offers the interest rate

[28] If it crosses zero, it must cross from below since $\pi_{0\ell}^B = -1$.

A1 Bank b would lend to some agents in the absence of securitization: its discounted return $\delta\rho E\left(S_\ell^B\right)$ from lending to the best agent (for whom $\eta\nu = 1$) exceeds the bank's unitary cost of capital. By equation (12), this implies that the deterring rate r_ℓ^{B*} is less than the gross project return ρ, so bank a lends at the deterring rate. Hence, by equations (12) and (13), bank a's credit score threshold under securitization is

$$\eta_\ell^B = \frac{\delta E\left(S_\ell^B\right) + \left(\Omega_{b\ell}^B\right)^+}{\delta E\left(S_\ell^B\right) + \Omega_{a\ell}^B}. \tag{14}$$

A2 Bank b benefits from securitization: $\Omega_{b\ell}^B > 0$.

A3 Bank a benefits more than bank b from securitization: $\Omega_{a\ell}^B > \Omega_{b\ell}^B$: Without this condition, $\eta_\ell^B \geq 1$, so bank a will not lend in the location.

In Figure 1, each agent in the location corresponds to a point in the unit square.[29] The agent's credit score η, which equals bank a's estimate of her type θ, appears on the horizontal axis. Bank b's estimate $\eta\nu$ of θ appears on the vertical axis.[30] While bank a sees only an agent's horizontal coordinate, bank b sees both.

In the absence of securitization, agents in areas A_0 and A_3 borrow from bank b at the interest rate ρ, while other agents do not get loans (Claim 4). With securitization, agents in areas A_3 and A_4 borrow from bank a at the deterring rate $r_\ell^{B*} < \rho$, while those in areas A_0 and A_1 get loans from bank b at the interest rate ρ.

5.1   The Effects of Securitization

A comparison of Claim 4 and Theorem 9 reveals the following effects of securitization, which are discussed in section 1.

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[29] The applicants are not uniformly distributed throughout the square.

[30] While the figure permits η and $\eta\nu$ each to take any value in the unit interval, some of these values may have zero probability.
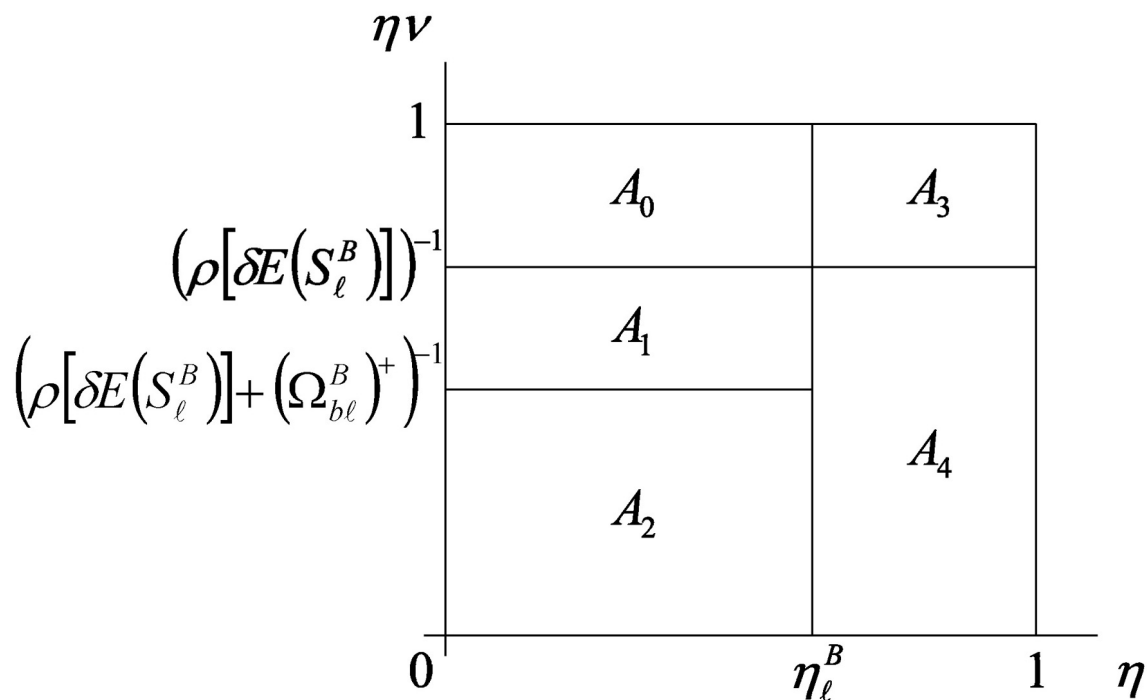
Figure 1: Effects of Securitization under Limit Irrelevance. A given location ℓ in region B is depicted. The credit rating η appears on the horizontal axis while bank b's estimate $\eta\nu$ of an applicant's type θ is depicted on the vertical axis. The figure assumes that $\delta\rho E\left(S_\ell^B\right) > 1$ and $\Omega_{a\ell}^B > \Omega_{b\ell}^B > 0$. Without securitization, applicants in regions A_0 and A_3 receive loans from bank b at the interest rate ρ. Those in regions A_1, A_2, and A_4 do not receive loans. With securitization, applicants in regions A_3 and A_4 receive loans from bank a at the interest rate $r_{\eta\ell}^{B*} < \rho$. Applicants in regions A_0 and A_1 receive loans from bank b at the interest rate ρ, while applicants in region A_2 do not receive loans.

1. **Securitization Stimulates Lending.** By connecting agents with liquid investors, securitization expands the set of borrowers.[31] In Figure 1, areas A_1 and A_4 are added.

2. **Securitization Favors Remote Lending.** Remote bank a lends to location ℓ only if it can securitize its loans.

3. **Remote Borrowers have Strong Observables but High Conditional Default Rates.** In Figure 1, the applicants who get remote loans are those whose credit scores η exceed bank a's threshold η_ℓ^B: they have strong observables. Now consider an otherwise identical neighborhood ℓ' in which the bank a's securitization profits $\Omega_{a\ell'}^B$ are higher than in location ℓ. This raises bank a's threshold: $\eta_{\ell'}^B > \eta_\ell^B$. The only applicants who are affected are those whose credit scores η lie between the two thresholds. In location ℓ, these applicants all get remote loans. In location ℓ' they get local loans, but only if bank b's estimate $\eta\nu$ of their type is at least $\left(\rho \left[\delta E \left(S_\ell^B \right) \right] + \left(\Omega_{b\ell}^B \right)^+ \right)^{-1} > 0$. Thus, ceteris paribus, a remote borrower with a given credit score η has an expected type $\eta\nu$ that is no higher, and sometimes strictly lower, than the expected type of a local borrower with the same credit score.

4. **Securitization Lets Borrowers with Strong Observables Get Cheap Remote Loans.** Securitization lowers the interest rate paid by agents with high credit scores (above η_ℓ^B) to $\min \left\{ \rho, r_\ell^{B*} \right\}$ while leaving unchanged the interest rate ρ paid by agents with lower credit scores.

5. **Securitization Raises Conditional and Unconditional Default Rates.** Securitization expands lending to a set of borrowers (in Figure 1, those in areas A_1 and A_4) whose expected types $\eta\nu$ are uniformly lower than those of agents

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[31] All effects described in sections 5.1 through 5.4 are intended in the weak sense: the set of borrowers weakly increases, etc. In the figures, these effects are strict.

who borrow without securitization (those in areas A_0 and A_3). This raises both conditional (on η) and unconditional default rates.

6. **Securitized Loans Have Higher Conditional Default Rates then Retained Loans.** For any given credit score η, securitized loans have higher mean default rates than retained loans. More precisely, let us compare two locations ℓ and ℓ' in region B. Assume bank b securitizes its loans to location ℓ' but not to location ℓ: $\Omega_{b\ell}^B < 0 < \Omega_{b\ell'}^B$. In all other respects, the two locations are identical. The comparison is depicted in Figure 2. For credit scores below η_ℓ^B, retained loans consist of area A_0 in location ℓ, while securitized loans consist of areas A_0 and A_1 in location ℓ. For each credit score, the securitized loans have a lower conditional expected type $\eta\nu$ than the retained loans. For credit scores above η_ℓ^B, all loans are securitized in both locations. Hence, for each credit score η for which there are retained loans in one location and securitized loans in the other, the latter group has a higher conditional default rate.

5.2 Higher Securitization Profits for the Local Bank

Suppose bank b's securitization profits rise from $\Omega_{b\ell}^B$ to $\widetilde{\Omega}_{b\ell}^B$. Since it is now harder to deter bank b from cream-skimming, bank a does so less often: it raises its credit score threshold from η_ℓ^B to $\widetilde{\eta}_\ell^B = \frac{\delta E(S_\ell^B) + (\widetilde{\Omega}_{b\ell}^B)^+}{\delta E(S_\ell^B) + \Omega_{a\ell}^B}$ (equation (14)). Theorem 9 implies the following effects, which are illustrated in Figure 3.

1. **Relatively More Local Lending.** Bank b lends more, while bank a lends less. In Figure 3, Bank b picks up borrowers in areas A_2 and A_5. Bank a stops lending to areas A_4 through A_6 and is left with only A_7 and A_8.

2. **More Lending to Diamonds in the Rough.** The set of borrowers grows to include those with credit scores below a's threshold η_ℓ^B, whose expected types lie between b's new and old thresholds. This is area A_2 in Figure 3. They are
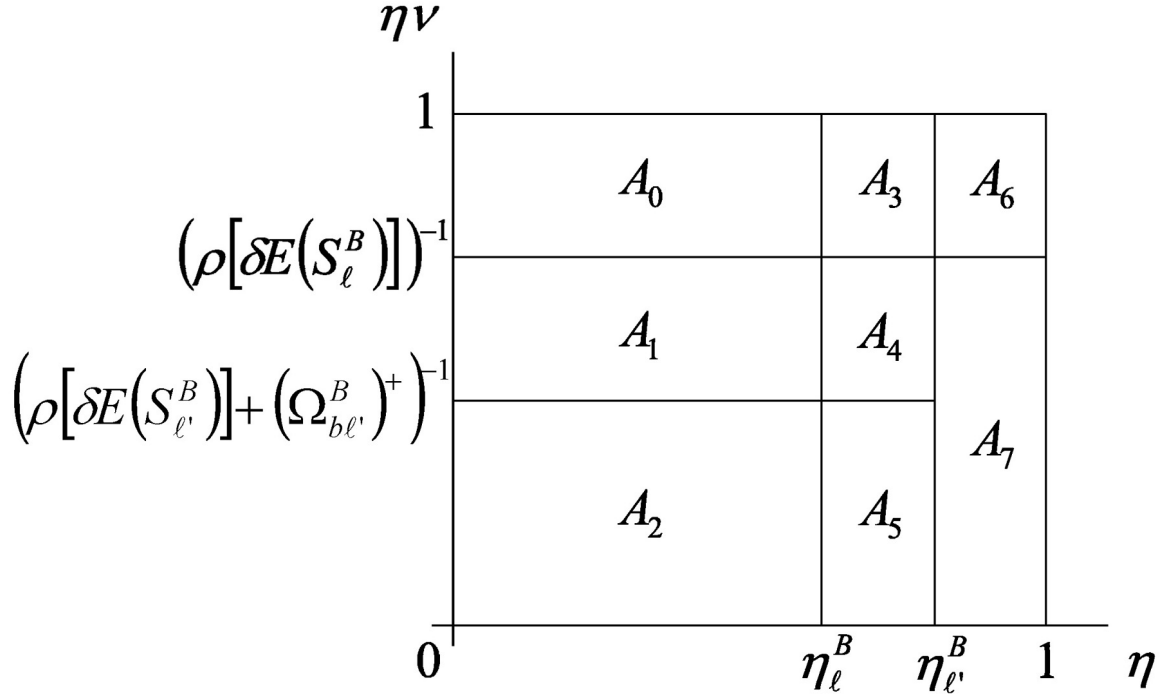
33

Figure 2: Retained Loans Have Lower Expected Default Rates. Two locations ℓ and ℓ' in region B are depicted. The credit rating η appears on the horizontal axis while bank b's estimate $\eta\nu$ of an applicant's type θ is depicted on the vertical axis. The figure assumes that $E\left(S_\ell^B\right) = E\left(S_{\ell'}^B\right)$, $\delta\rho E\left(S_\ell^B\right) > 1$, $\Omega_{a\ell}^B = \Omega_{a\ell'}^B > 0$, and $\Omega_{b\ell}^B < 0 < \Omega_{b\ell'}^B$. For credit scores below η_ℓ^B, retained loans consist of area A_0 in location ℓ, while securitized loans consist of areas A_0 and A_1 in location ℓ. For credit scores above η_ℓ^B, all loans are securitized in both locations. Hence, for each credit score η, retained loans (if there are any) have a higher expected type $\eta\nu$ than securitized loans.
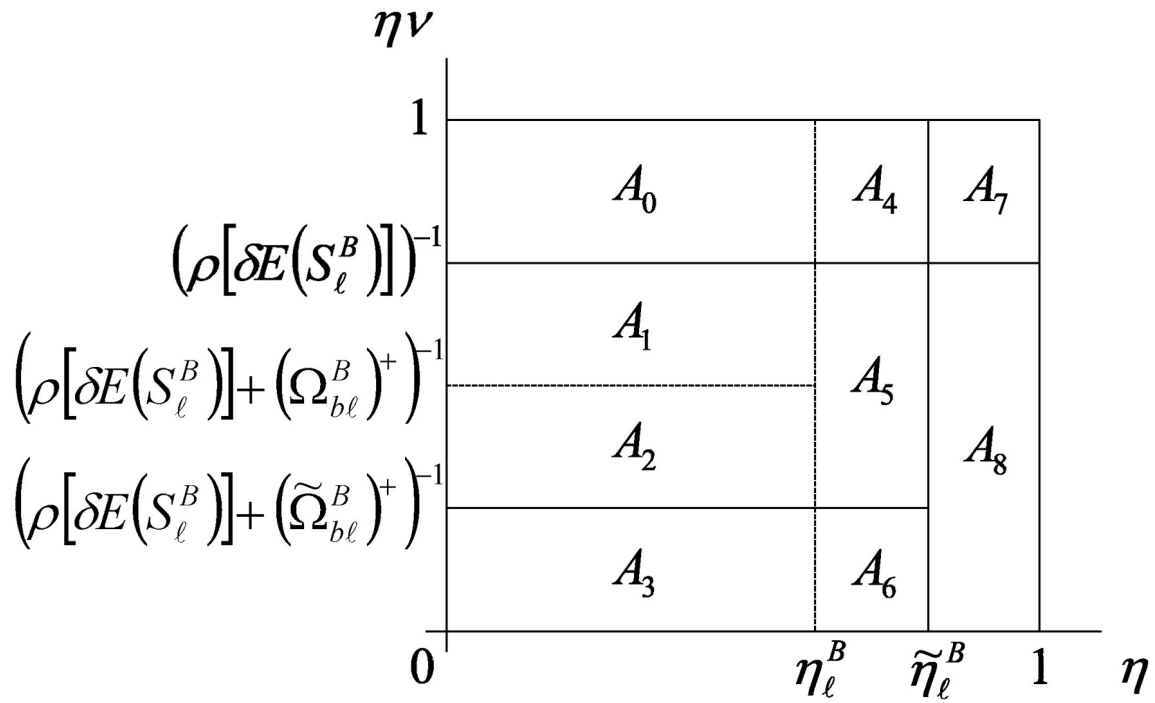
$\eta\nu$

1

$\left(\rho\left[\delta E\left(S_{\ell}^{B}\right)\right]\right)^{-1}$

$\left(\rho\left[\delta E\left(S_{\ell}^{B}\right)\right]+\left(\Omega_{b\ell}^{B}\right)^{+}\right)^{-1}$

$\left(\rho\left[\delta E\left(S_{\ell}^{B}\right)\right]+\left(\widetilde{\Omega}_{b\ell}^{B}\right)^{+}\right)^{-1}$

A_0 $\qquad$ A_4 $\quad$ A_7

A_1

A_5

A_2

A_8

A_3 $\qquad$ A_6

0 $\qquad$ η_{ℓ}^{B} $\quad$ $\widetilde{\eta}_{\ell}^{B}$ $\quad$ 1 $\quad$ η

Figure 3: Effect of Increase in Bank b's Securitization Profits from $\Omega_{b\ell}^{B}$ to $\widetilde{\Omega}_{b\ell}^{B}$. The conditions of Figure 1 are assumed to hold before and after the increase, which raises bank a's threshold from η_{ℓ}^{B} to $\widetilde{\eta}_{\ell}^{B}$. Bank a, which initially lent to areas A_4 through A_8, now only lends to areas A_7 and A_8 and charges a lower interest rate to this group. Bank b adds areas A_2, A_4, and A_5 to its initial borrower pool of A_0 and A_1.

diamonds in the rough: while their credit scores lie below bank a's threshold, their expected types are the highest among those who previously did not borrow.

3. **Welfare Transfer from Good to Great Borrowers** (in terms of observables). As bank a no longer competes for agents with credit scores between η_ℓ^B and $\widetilde{\eta}_\ell^B$, their interest rate rises from $\min\left\{\rho, r_\ell^{B*}\right\}$ to ρ. However, those with scores above $\widetilde{\eta}_\ell^B$ see their interest rate fall since the rate bank a must offer to deter bank b is now lower (equation (12)).

5.3 Higher Securitization Profits for the Remote Bank

Theorem 9 implies the following the effects of an increase in bank a's securitization profits from $\Omega_{a\ell}^B$ to $\widehat{\Omega}_{a\ell}^B$. These are illustrated in Figure 4, where $\widehat{\eta}_\ell^B$ denote bank a's new, lower credit score threshold.

1. **Relatively More Remote Lending.** Bank a lends more, while bank b lends less. In Figure 4, a picks up borrowers in areas A_3 through A_5, while b stops lending to areas A_3 and A_4 and is left with only A_0 and A_1.

2. **Applicants with High Credit Scores Benefit from More Loans.** The set of borrowers grows to include those with credit scores between bank a's old and new thresholds (area A_5 in the figure). Among agents who initially did not get loans, these borrowers have the highest credit scores. These agents benefit since their interest rate, $\min\left\{\rho, r_\ell^{B*}\right\}$, is lower than the project return ρ.

5.4 Efficiency Effects

We next turn to the efficiency effects of securitization. In order for loans to be allocated efficiently *within* each location, a resident of location ℓ in region R must get a loan if and only if her expected project return $\rho\eta\nu E\left(S_\ell^R\right)$ exceeds a location-specific threshold c_ℓ^R. In order for the allocation also to be efficient *across* locations
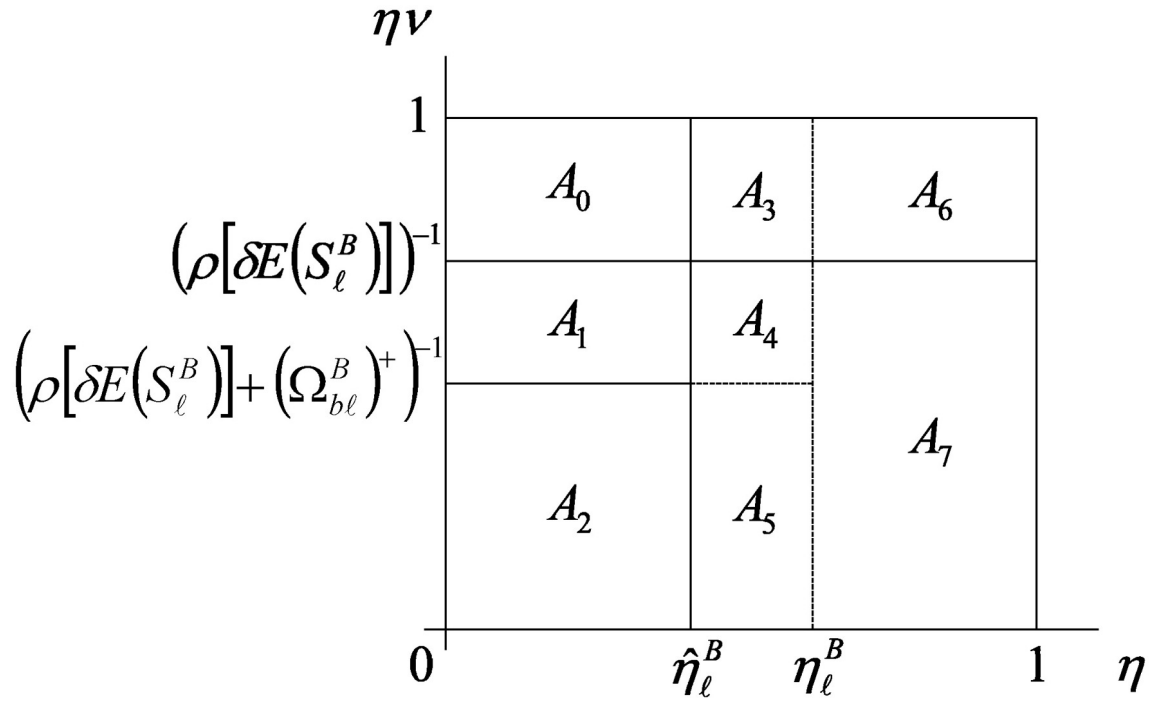
36

Figure 4: Effect of Increase in Bank a's Securitization Profits from $\Omega_{a\ell}^{B}$ to $\widehat{\Omega}_{a\ell}^{B}$. The conditions of Figure 1 are assumed to hold before and after the increase, which lowers bank a's threshold from η_{ℓ}^{B} to $\widehat{\eta}_{\ell}^{B}$. Bank b ceases to lend to areas A_3 and A_4 and now only lends to areas A_0 and A_1. Bank a adds areas A_3 through A_5, and A_5 to its initial borrower pool of A_6 and A_7. There is no change in the interest rates offered by the two banks.

and regions, this threshold must not depend on the location ℓ or region R. This is true without securitization, where the threshold c_ℓ^R equals δ^{-1} (Claim 4).

It is useful to restate the condition for within-location efficiency as follows: an agent gets a loan if and only if her expected type $\eta\nu$ exceeds some location-specific threshold $\widetilde{c}_\ell^R$.[32] This holds without securitization, where only agents in areas A_0 and A_3 get loans. However, with securitization it fails, since the threshold is zero if an agent's credit score exceeds η_ℓ^B and $r_\ell^{B*}/\rho > 0$ otherwise.

This discussion reveals two types of inefficiencies that are caused by securitization.

1. **Public Information Bias.** Since bank a relies exclusively on public signals to screen agents, there is an inefficient bias towards agents whose public information is strong. In Figure 1, agents near the top of area A_2, who are turned down by both banks, are of higher quality than agents near the bottom of area A_4, who get loans from bank a.

2. **Securitization Profit Bias.** Efficiency requires that a bank consider only an agent's creditworthiness. However, in equilibrium a bank also prefers agents who are more profitable to securitize. For instance, we can reinterpret Figure 3 as comparing two locations in region B, in which bank b's securitization profits are $\Omega_{b\ell}^B$ and the higher value $\widetilde{\Omega}_{b\ell}^B$, respectively. Agents in the top of A_2 in the former location are turned down, while agents in the bottom of the same area in the latter region receive funding. In Figure 4, agents at the top of area A_5 are turned down when bank a's securitization profits are $\Omega_{a\ell}^B$ while agents at the bottom of the same area receive loans when these profits take the higher value $\widehat{\Omega}_{a\ell}^B$. In both cases, efficiency requires the opposite.

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[32]In particular, $\widetilde{c}_\ell^R$ equals $c_\ell^R \left[\rho E \left(S_\ell^R \right) \right]^{-1}$.

6 Related Literature

While prior models have studied the interaction between a single bank's securitization and lending decisions, ours appears to be the first to study the effect of securitization on lending *competition*. We now discuss the relations between our work and this prior research, as well as related work on security design and on lending competition under adverse selection.

6.1 Lending with Securitization

Bubb and Kaufman [6] (BK) study a model with a single bank and a continuum of loan applicants. The bank sees each applicant's credit score. It can also engage in costly screening, which reveals soft information about the applicant. Without securitization, the bank lends to applicants with high scores and rejects those with weak ones. It screens applicants with intermediate scores and lends to them if and only if their soft information is positive. BK then introduce a monopsony loan buyer. The buyer commits to buying a smaller fraction of loans to intermediate borrowers in order to ensure that the bank will still screen them. In contrast, our model has many small and unorganized security buyers, so such commitment is impossible. Rather, a bank lends remotely in order to have less private information when it issues its security. As a bank cannot discover a remote applicant's soft information, remote lending raises the default risk. In contrast, securitization does not raise defaults in BK.

Hartman-Glaser, Piskorski, and Tchistyi [17] study the optimal design of mortgage-backed securities by a lender who can exert costly hidden effort to screen loan applicants. The model takes place in continuous time. Loans default according to a Poisson process. The lender can lower the arrival rate of defaults at a cost. The model is aspatial and features fixed loan terms and a single bank. In contrast, ours is a spatial model with endogenous interest rates and two competing banks. While

they study moral hazard, our focus is adverse selection.

Heuson, Passmore, and Sparks [19] (HPS) study a model in which applicants have a continuum of publicly observable default probabilities. A bank chooses whether to lend to an applicant and, if so, whether to securitize the loan. An investor then sets the minimum such probability to accept a loan for securitization. In response, the bank retains the best loans, securitizes intermediate loans, and doesn't lend to the worst borrowers. This mirrors the behavior of a bank towards its local applicants in our model. While HPS study the problem of a single bank under symmetric information, we assume two banks who face asymmetric information at both the lending and securitization stages.

Chemla and Hennessy [7] (CH) also study a model of lending with securitization. A bank can exert costly effort to raise the chance that its loans will have a high return. There are three types of investors. The first group are uninformed risk-averse hedgers for whom the bank's security is a utility-enhancing hedge against future endowment risk. CH offer the example of future home buyers: when the economy booms, few borrowers default on their mortgages, so the security has a high payoff; but the boom also raises house prices, so investors need more money. The security thus hedges against housing market risk. There is also a wealthy, risk-neutral speculator who sees a signal of the asset's type and can exert costly effort to increase the precision of this signal. Finally, there is a continuum of risk-neutral "market makers" with deep pockets.

For some parameters, the model has a pooling equilibrium in which the bank always securitizes all of its loans. It issues a senior tranche as well as an equity-like mezzanine tranche that is attractive to the hedgers. The hedgers' demand stimulates information acquisition by the speculator, since he can profit from the hedgers' ignorance. The resulting informed speculation increases the correlation of the security price with its true value, which gives the bank an ex ante incentive to screen. This incentive can actually be stronger than in the separating equilibrium in which the bank

issues more shares when quality is low. Thus, CH argue that securitization without retention does not necessarily worsen the moral hazard problem, since tranching can lead to informative prices that give the bank an incentive to screen.

Shleifer and Vishny [33] analyze a model in which banks have private information about loan quality (which is either high or low) and must retain a fixed fraction of the loan if they sell it. Loans are sold individually. Security prices are affected by investor sentiment. Since they assume symmetric information with irrational investors, their model bears little relation to ours.

6.2   Security Design

Our paper is closely related to DeMarzo and Duffie [10] (DD). They study the problem of a risk-neutral issuer who has a fixed portfolio of long term assets. The issuer designs a single security, which consists of a portfolio of assets to securitize and a payoff function: a map from the final value of this portfolio to the security's payoff. The issuer then sees a private signal of the portfolio's value and chooses a proportion of the security to offer for sale to a continuum of uninformed, risk-neutral investors who are more patient than the issuer. There is a unique separating equilibrium. When the issuer's signal is higher, it sells a lower proportion of the security and the market responds with a higher price.

Signalling is costly since the issuer sells less of the security when the gains from trade are greater. For this reason, the issuer's goal at the design stage is to minimize the sensitivity of its security's payoff to its private information. DD show that within the class of monotone, limited-liability securities, this sensitivity is minimized by debt.[33] Intuitively, debt pays its fixed face value when the value of its underlying portfolio exceeds this value. If the debt defaults, it pays the value of its underlying portfolio, which is as close to its face value as limited liability will allow. Hence,

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[33]A security is monotone and limit-liability if its payoff function $\varphi : \Re_+ \to \Re_+$ is nondecreasing and satisfies $\varphi(y) \in [0, y]$ for each realization y of the final value of the portfolio of securitized assets.

the payoff function of debt is as flat as possible within the class of monotone, limited liability payoff functions.[34]

In DD, the issuer's initial asset portfolio is taken as given. The theoretical contribution of our paper is to derive this portfolio as the endogenous result of lending competition. We assume two regional banks who compete for borrowers. The outcome of this competition determines each bank's loan portfolio, which it then securitizes as in DD. Each bank has private information about local applicants at the lending stage. This gives local banks an advantage in competing for loans. Each bank also observes a private signal of its local macroeconomic conditions prior to issuing its security. This creates a lemons problem that favors remote lending.

Since a bank's security can contain a mixture of local and remote loans, a bank's macroeconomic signal contains information about the value of the other bank's security. Hence, the quantity that a bank chooses to sell acts as a signal of the values of both banks' securities. Nevertheless, DD's single-issuer result generalizes: there is a unique separating equilibrium, in which each bank's payoff is the same as in the single-issuer case. Moreover, each bank issues debt. We use this result to derive rich implications for the composition of each bank's loan portfolio.

Like DD, we assume each bank issues at most one security. In contrast, DeMarzo [9] studies the case of a risk-neutral issuer who designs one or more securities based on a finite, exogenous set of assets. The issuer then sees signals of the final values of its assets and chooses how much of each security to sell. DeMarzo shows that pooling the assets before designing the security has a cost and a benefit for the issuer. The cost is information destruction: pooling prevents the issuer from signalling positive

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[34]Monotonicity is needed since the issuer's signal is noisy. In particular, suppose the bank issues a security that behaves like debt with one exception: its payoff falls slightly in particularly good states. Assume these states have positive probability for intermediate signal values as well. Then this change might lead to a smaller rise in the estimated security payoff as the issuer's signal rises from low to intermediate values. Hence, this security might be even less informationally sensitive than debt.

information for some securities and negative information for others. The benefit is diversification: if the assets' final values conditional on the signals are not perfectly correlated, then pooling them lowers the risk of security default. Whether pooling is optimal depends on whether the diversification benefit outweighs the information destruction effect.

Permitting multiple securities would have two effects in our model. First, a bank's profits from securitizing its loans to a given location would depend on which of its various securities it would optimally add the loans to. Second, in the issuance game between the two banks, each bank would choose multiple quantities rather than a single quantity. It seems unlikely that either of these changes would alter our basic results. For simplicity, therefore, we follow DD in restricting each bank to a single security.

Another way banks generate multiple securities is to issue multiple tranches of a single loan portfolio. A bank may also be able to delay designing its security until after it discovers its private information. DeMarzo [9] shows that these practices are equivalent.[35] While DD's [10] securitization profit function has a closed form solution, DeMarzo's [9] profit function depends on the solution to a differential equation. This makes it challenging to incorporate into our setting. However, the two functions have some properties in common (DeMarzo [9, Lemmas 5 and 9]), so some of our findings might generalize. This might be an interesting question for future research.

Adverse selection in security issuance was first analyzed by Myers and Majluf [26]. They assume a firm must raise a fixed amount of capital and focus on equity issuance, while briefly considering debt. Nachman and Noe [27] (NN) also assume a fixed amount of capital must be raised but allow for a full set of securities. They give distributional conditions that are sufficient for a firm to issue debt. In their

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[35]More precisely, delaying security design until after the (one dimensional) information is revealed is equivalent to issuing an unlimited number of tranches (each of which has a monotone payoff) before the information is revealed.

work, the security is designed ex post, while DD assume ex ante design. Axelson [2] reverses the usual informational assumptions: investors are informed while the issuer is not. Like DD and NN, he finds that debt is optimal.

Biais and Mariotti [3] (BM) modify DD in two essential ways. They assume that security buyers have market power and thus earn positive profits. Moreover, they use mechanism design to analyze the optimal trading mechanism, while DD assume that it is a signalling game. BM also find that the optimal security is debt. However, in BM's optimal trading mechanism, all issuer types sell 100% of their securities. This contrasts with DD, in which there is some retention.

Boot and Thakor [4] analyze a model in which a firm has various assets that it wishes to sell, and investors can exert costly effort to discover information about these assets' values. There are noise traders, so gathering information can be profitable. Splitting the firm's assets into two securities, one informationally sensitive and the other not, stimulates trade, which gives investors an incentive to discover information about the assets' values. This is profitable for the issuer since it mitigates adverse selection. The results of Chemla and Hennessy [7], discussed above, build on this insight.

Demange and Laroque [11] and Rahi [29] study models in which a risk-averse entrepreneur with a noisy private signal of the value of his projects designs and sells securities. In these papers, unlike DD and ours, the issuer decides how much to issue before observing her private information. The private information only permits the issuer to earn trading profits afterwards.

6.3   Lending Competition with Adverse Selection

Our model is also related to prior research on lending competition with adverse selection in the absence of securitization. Perhaps the closest is Hauswald and Marquez [18] (HM). HM assume that a bank's cost of gathering soft information is greater for more distant applicants. This is also true in our model, where the cost is zero

for local applicants and infinite for remote ones. Because banks know more about local applicants, they lend at high interest rates to quality local applicants and offer low interest rates to some remote applicants. In HM, the latter effect occurs because other banks - fearing a winner's curse less - compete more aggressively for these remote applicants. In our model, it is because offering a lower interest rate prevents cream-skimming by a remote applicant's local bank. In both models, remote borrowers default more since their lending banks have less information about their credit quality.

In an earlier model, Sharpe [31] assumes that a bank's soft information arises endogenously from its prior loans to applicants. Because of a winner's curse, banks that lack this information do not lend to mature applicants. Analogously, in our model all lending is local if banks cannot securitize. Finally, in Broecker [5], each bank sees a noisy private signal of each loan applicant's type. Since a bank attracts only those borrowers who are turned down by banks that offer lower rates, a bank that charges a high rate tends to get low quality applicants. Similarly, remote banks in our model charge low interest rates in order to avoid cream-skimming by better informed local banks.

7 Conclusions

The model of DeMarzo and Duffie [10] assumes a single issuer who designs a single security. The issuer then sees private information about this security's value and chooses how much to sell. In equilibrium, the issuer varies the amount that it sells in order to signal the security's value. This is costly for the issuer since it must sell less of the security when the gains from trade are higher. In order to minimize these costs, the issuer designs a security that is not very sensitive to its private information.

In DeMarzo and Duffie [10], the issuer's initial portfolio of assets is exogenous. This is an important limitation: in practice, a bank's lending behavior may be

influenced by its expected profits from securitizing its resulting loan portfolio. We study this issue in a rich setting in which regional banks first compete for borrowers and then design and issue securities based on their resulting loan portfolios.

As in prior models, we find that securitization expands lending by connecting liquid investors with loan applicants. However, we also find that securitization creates a bias towards remote loans, which can be securitized without contributing to a bank's lemons problem. Moreover, since banks lack soft information about remote applicants, remote borrowers tend to have stronger observables than local borrowers. In addition, banks must offer lower interest rates to remote applicants in order to prevent cream skimming by the applicants' local banks. Thus, remote loans will have lower interest rates than local loans, and securitization strengthens the negative relation between a borrower's public information and the interest rates she pays.

Since banks lack soft information about remote applicants, they do not screen as well when lending remotely. Hence securitization, which stimulates remote lending, raises borrowers' conditional and unconditional default rates. Moreover, in cross section, securitized loans will have higher default rates conditional on observables since banks lower lending standards more in local areas that are more profitable to securitize. As detailed in section 1, all of our predictions are consistent with prior empirical research.

While securitization has the potential to raise social welfare by connecting liquid investors with worthy loan applicants, this is tempered by two inefficiencies. The first is public information bias: since the remote bank relies exclusively on observables, there is an inefficient bias towards applicants with strong observables such as credit scores. This is inefficient as these applicants are favored over creditworthy applicants with weak observables.

The second inefficiency is securitization profit bias. Efficiency requires that only the most creditworthy applicants get loans. However, with securitization banks also prefer applicants who enhance the value of their security. One reason can be that the

bank is not well acquainted with the applicant's local macroeconomic environment. In this case, the applicant's loan does not add much to the lemons problem the bank will face in selling its security. Another is that the applicant has a good chance of repaying her loan in bad macroeconomic states. By raising the payout to investors when the security defaults, these borrowers raise the security's value, which translates into greater securitization profits. However, since all participants are risk neutral, it is inefficient to favor these borrowers.

A   Proofs

Proof of Claim 1:   Let $\overline{\eta}_\ell = \sup_{s_{\mathrm{pub}}} \eta\left(s_{\mathrm{pub}}|\ell\right)$, $\underline{\eta}_\ell = \inf_{s_{\mathrm{pub}}} \eta\left(s_{\mathrm{pub}}|\ell\right)$, $\underline{\nu}_{s_{\mathrm{pub}},\ell} = \inf_{s_{\mathrm{priv}}} \nu\left(s_{\mathrm{priv}}|s_{\mathrm{pub}},\ell\right)$, and $\overline{\nu}_{s_{\mathrm{pub}},\ell} = \sup_{s_{\mathrm{priv}}} \nu\left(s_{\mathrm{priv}}|s_{\mathrm{pub}},\ell\right)$. Integrating by parts,

$$\eta\left(s_{\mathrm{pub}}|\ell\right) = \int_\theta \theta dF\left(\theta|s_{\mathrm{pub}},\ell\right) =$$

$$-\int_\theta \theta d\left(1 - F\left(\theta|s_{\mathrm{pub}},\ell\right)\right) = \int_\theta \left(1 - F\left(\theta|s_{\mathrm{pub}},\ell\right)\right) d\theta.$$

Hence, $\frac{\partial}{\partial s_{\mathrm{pub}}} \eta\left(s_{\mathrm{pub}}|\ell\right)$ equals $-\int_\theta \frac{\partial F\left(\theta|s_{\mathrm{pub}},\ell\right)}{\partial s_{\mathrm{pub}}} d\theta$ which, by Public Signal Monotonicity, exists and lies in $\left[\int_\theta \underline{\lambda}\left(\theta\right) d\theta, \ \int_\theta \overline{\lambda}\left(\theta\right) d\theta\right]$. In addition,

$$E\left(\theta|s_{\mathrm{priv}}, s_{\mathrm{pub}}, \ell\right) = \int_\theta \theta dF\left(\theta|s_{\mathrm{priv}}, s_{\mathrm{pub}}, \ell\right) =$$

$$-\int_\theta \theta d\left(1 - F\left(\theta|s_{\mathrm{priv}}, s_{\mathrm{pub}}, \ell\right)\right) = \int_\theta \left(1 - F\left(\theta|s_{\mathrm{priv}}, s_{\mathrm{pub}}, \ell\right)\right) d\theta.$$

Thus, $\frac{\partial}{\partial s_{\mathrm{priv}}} E\left(\theta|s_{\mathrm{priv}}, s_{\mathrm{pub}}, \ell\right) = -\int_\theta \frac{\partial F\left(\theta|s_{\mathrm{priv}}, s_{\mathrm{pub}}, \ell\right)}{\partial s_{\mathrm{priv}}} d\theta$ which, by Private Signal Monotonicity, exists and lies in $\left[\int_\theta \underline{\mu}\left(\theta\right) d\theta, \ \int_\theta \overline{\mu}\left(\theta\right) d\theta\right]$. Since $\eta \in (0,1)$, the slope of $\nu\left(s_{\mathrm{priv}}|s_{\mathrm{pub}}, \ell\right)$ lies in $\Re_{++}$. Q.E.D.

Proof of Claim 2:   The proof of Claim 1 implies that (a) $\eta\left(s_{\mathrm{pub}}|\ell\right)$ has a differentiable inverse function $s_{\mathrm{pub}}\left(\eta|\ell\right)$ of η, which is a bijection from $\left(\underline{\eta}_\ell, \overline{\eta}_\ell\right) \subset [0,1]$ to $(0,1)$ whose slope lies in $\Re_{++}$ and (b) $\nu\left(s_{\mathrm{priv}}|s_{\mathrm{pub}}, \ell\right)$ has a differentiable inverse function $s_{\mathrm{priv}}\left(\nu|s_{\mathrm{pub}}, \ell\right)$ of ν, which is a bijection from $\left(\underline{\nu}_{s_{\mathrm{pub}},\ell}, \overline{\nu}_{s_{\mathrm{pub}},\ell}\right) \subset \left[0, \eta\left(s_{\mathrm{pub}}|\ell\right)^{-1}\right]$

to $(0,1)$ whose slope lies in $\Re_{++}$. We now extend the function $s_{\mathrm{pub}}(\eta|\ell)$ to all pairs (η,ℓ) in $(0,1)\times[0,1]$ by defining it as one if $\eta > \sup_{s_{\mathrm{pub}}} \eta\,(s_{\mathrm{pub}}|\ell)$ and zero if $\eta < \inf_{s_{\mathrm{pub}}} \eta\,(s_{\mathrm{pub}}|\ell)$. By Claim 1,

$$G(\eta_0,\ell_0) = \Pr\left(s_{\mathrm{pub}} < s_{\mathrm{pub}}(\eta_0|\ell) \text{ and } \ell \leq \ell_0\right) = \int_{\ell=0}^{\ell_0}\int_{s_{\mathrm{pub}}=-\infty}^{s_{\mathrm{pub}}(\eta_0|\ell)} f\,(s_{\mathrm{pub}},\ell)\,ds_{\mathrm{pub}}d\ell.$$

Hence, $g(\eta_0,\ell_0) = \frac{\partial^2 G(\eta_0,\ell_0)}{\partial\eta_0\partial\ell_0} = f\,(s_{\mathrm{pub}}(\eta_0|\ell_0),\ell_0)\frac{\partial s_{\mathrm{pub}}(\eta_0|\ell_0)}{\partial\eta_0}$ where $f\,(s_{\mathrm{pub}}(\eta_0|\ell_0),\ell_0)$ denotes the marginal density $f\,(s_{\mathrm{pub}},\ell)$ evaluated at $(s_{\mathrm{pub}}(\eta_0|\ell_0),\ell_0)$. Since (η_0,ℓ_0) is feasible, $\inf_{s_{\mathrm{pub}}} \eta\,(s_{\mathrm{pub}}|\ell_0) < \eta_0 < \sup_{s_{\mathrm{pub}}} \eta\,(s_{\mathrm{pub}}|\ell_0)$, whence $s_{\mathrm{pub}}(\eta_0|\ell_0)$ lies in $(0,1)$ by Claim 1. Thus, $f\,(s_{\mathrm{pub}}(\eta_0|\ell_0),\ell_0)\in\Re_{++}$ by assumption. Claim 1 implies further that $\frac{\partial s_{\mathrm{pub}}(\eta_0|\ell_0)}{\partial\eta_0}\in\Re_{++}$. Thus, $g(\eta_0,\ell_0)\in\Re_{++}$ as claimed. Q.E.D.

Proof of Claim 3: First,

$$
\begin{aligned}
H(\nu_0|\eta_0,\ell_0) &= \Pr\left(\nu\,(s_{\mathrm{priv}}|s_{\mathrm{pub}},\ell)\leq\nu_0|s_{\mathrm{pub}}=s_{\mathrm{pub}}(\eta_0|\ell_0),\ell=\ell_0\right)\\
&= \Pr\left(s_{\mathrm{priv}}\leq s_{\mathrm{priv}}(\nu_0|s_{\mathrm{pub}},\ell)\,|s_{\mathrm{pub}}=s_{\mathrm{pub}}(\eta_0|\ell_0),\ell=\ell_0\right)\\
&= F\left(s_{\mathrm{priv}}(\nu_0|s_{\mathrm{pub}},\ell)\,|s_{\mathrm{pub}}(\eta_0|\ell_0),\ell_0\right).
\end{aligned}
$$

Hence,

$$
\begin{aligned}
H(\nu|\eta,\ell) &= F\left(s_{\mathrm{priv}}(\nu|s_{\mathrm{pub}},\ell)\,|s_{\mathrm{pub}}(\eta|\ell),\ell\right),\\
H'(\nu|\eta,\ell) &= F'\left(s_{\mathrm{priv}}(\nu|s_{\mathrm{pub}},\ell)\,|s_{\mathrm{pub}}(\eta|\ell),\ell\right)s'_{\mathrm{priv}}(\nu|s_{\mathrm{pub}},\ell),\text{ and}\\
H''(\nu|\eta,\ell) &= F''\left(s_{\mathrm{priv}}(\nu|s_{\mathrm{pub}},\ell)\,|s_{\mathrm{pub}}(\eta|\ell),\ell\right)\left[s'_{\mathrm{priv}}(\nu|s_{\mathrm{pub}},\ell)\right]^2\\
&\quad +F'\left(s_{\mathrm{priv}}(\nu|s_{\mathrm{pub}},\ell)\,|s_{\mathrm{pub}}(\eta|\ell),\ell\right)s''_{\mathrm{priv}}(\nu|s_{\mathrm{pub}},\ell).
\end{aligned}
$$

Differentiating the identity $\nu\left(s_{\mathrm{priv}}(\nu|s_{\mathrm{pub}},\ell)\,|s_{\mathrm{pub}},\ell\right)=\nu$ with respect to ν,

$$
\begin{aligned}
1 &= \nu'\left(s_{\mathrm{priv}}(\nu|s_{\mathrm{pub}},\ell)\,|s_{\mathrm{pub}},\ell\right)s'_{\mathrm{priv}}(\nu|s_{\mathrm{pub}},\ell)\text{ and}\\
0 &= \nu'\left(s_{\mathrm{priv}}(\nu|s_{\mathrm{pub}},\ell)\,|s_{\mathrm{pub}},\ell\right)s''_{\mathrm{priv}}(\nu|s_{\mathrm{pub}},\ell)\\
&\quad +\nu''\left(s_{\mathrm{priv}}(\nu|s_{\mathrm{pub}},\ell)\,|s_{\mathrm{pub}},\ell\right)\left[s'_{\mathrm{priv}}(\nu|s_{\mathrm{pub}},\ell)\right]^2
\end{aligned}
$$

so that

$$
\begin{aligned}
s'_{\mathrm{priv}}\left(\nu|s_{\mathrm{pub}},\ell\right) &= \left[\nu'\left(s_{\mathrm{priv}}\left(\nu|s_{\mathrm{pub}},\ell\right)|s_{\mathrm{pub}},\ell\right)\right]^{-1} = \left[\nu'\left(s_{\mathrm{priv}}|s_{\mathrm{pub}},\ell\right)\right]^{-1} \text{ and} \\
s''_{\mathrm{priv}}\left(\nu|s_{\mathrm{pub}},\ell\right) &= -\frac{\nu''\left(s_{\mathrm{priv}}|s_{\mathrm{pub}},\ell\right)}{\left[\nu'\left(s_{\mathrm{priv}}|s_{\mathrm{pub}},\ell\right)\right]^3}.
\end{aligned}
$$

Accordingly,

$$
\frac{H''\left(\nu|\eta,\ell\right)\nu}{H'\left(\nu|\eta,\ell\right)} = \frac{\left(F''\left[s'_{\mathrm{priv}}\right]^2 + F's''_{\mathrm{priv}}\right)\nu}{F's'_{\mathrm{priv}}} = \frac{F''}{F'\nu'} - \frac{\nu''\nu}{\left[\nu'\right]^2}.
$$

Q.E.D.

Proof of Claim 4: Consider, for instance, bank a. If it competes for a borrower in region B, it must make the first offer. It does not know the borrower's private type ν. Since bank b knows ν, bank b can tell which of bank a's offers are profitable for bank a and which are not. However, in the absence of securitization the two banks have common values: the value of lending to a borrower is simply her discounted expected repayment less the common cost of capital. Thus, bank b will slightly underbid bank a's profitable offers and refrain from bidding on the unprofitable ones. As a result, bank a will succeed in lending only to unprofitable borrowers. Knowing this, bank a will not make offers to any agents who reside in region B in period 1. But given this, in period 2 bank b can charge the maximum possible interest rate of ρ and any of these agents will agree. It will do so if the resulting discounted expected repayment, $\delta\rho\eta\nu E\left(S_\ell^R\right)$, exceeds its unitary cost of capital. Q.E.D.

Proof of Claim 6: Equilibrium requires that bank a does not want to change its strategy taking bank b's strategy q_b as given. Define

$$
P_a\left(q,\sigma\right) = \int_{u^b \in \Re_+^N} p_a\left(q, q_b\left(u^b\right), \sigma\right) d\Psi\left(u^b|\sigma\right)
$$

and

$$
\Phi_a\left(u^a,\sigma\right) = \int_{u^b \in \Re_+^N} \phi_a\left(u^a, u^b, \sigma\right) d\Psi\left(u^b|\sigma\right).
$$

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Integrating conditions 1 and 3 over bank b's possible signal vectors u^b, we find that, almost surely, bank a's optimal quantity q maximizes $q\left[P_a\left(q,\sigma\right) - \delta\Phi_a\left(u^a,\sigma\right)\right]$ and $P_a\left(q_a\left(u^a,\sigma\right),\sigma\right)$ equals $\Phi_a\left(u^a,\sigma\right)$. These are the conditions for a separating equilibrium of the single-sender game analyzed by DeMarzo and Duffie [10, p. 77]. By their Proposition 2 [10, p. 78], bank a sells a quantity $q_a\left(u^a,\sigma\right) = \left[\widehat{\phi}_a\left(0,\sigma\right)/\widehat{\phi}_a\left(u^a,\sigma\right)\right]^{\frac{1}{1-\delta}}$ and the expected market price when bank a sells a quantity q is $P_a\left(q,\sigma\right) = \widehat{\phi}_a\left(0,\sigma\right)/q^{1-\delta}$. The quantity and expected price of bank a's security thus does not depend on bank b's strategy since $\widehat{\phi}_a\left(u^a,\sigma\right)$ does not. Hence, DeMarzo and Duffie's equation (4) [10, p. 79] implies that in any separating equilibrium, bank a's securitization profits conditional on the signals u^a and σ are given by $\pi\left(u^a,\sigma\right) = \left(1-\delta\right)\widehat{\phi}_a\left(0,\sigma\right)^{\frac{1}{1-\delta}}\widehat{\phi}_a\left(u^a,\sigma\right)^{-\frac{\delta}{1-\delta}}$ as claimed.

It remains to show that bank a's optimal security is debt. Following DeMarzo and Duffie [10, pp. 88-89], let $\varphi_a\left(\cdot\right)$ be any monotone security. Since Y_a is nondecreasing in each factor ζ_k^A, for each public signal σ the lowest possible realization of $E\left(\varphi_a\left(Y_a\right)|u^a,\sigma\right)$ is $E\left(\varphi_a\left(Y_a\right)|u^a = 0;\sigma\right)$. Now consider a standard debt security $\min\left\{m_a,Y_a\right\}$. By the dominated convergence theorem, $E\left(\min\left\{m_a,Y_a\right\}|u^a = 0;\sigma\right)$ is continuous in m_a, so we may choose m_a so that $E\left(\min\left\{m_a,Y_a\right\}|u^a = 0;\sigma\right) = E\left(\varphi_a\left(Y_a\right)|u^a = 0;\sigma\right)$. Let $d\left(Y_a\right) = \varphi_a\left(Y_a\right)-\min\left\{m_a,Y_a\right\}$ and $\psi\left(u^a,\sigma\right) = E\left(d\left(Y_a\right)|u^a,\sigma\right)$; by construction, $\psi\left(0,\sigma\right) = 0$. Because $\varphi_a\left(Y_a\right) \leq Y_a$, for $Y_a \leq m_a$ we have $d\left(Y_a\right) = \varphi_a\left(Y_a\right) - Y_a \leq 0$. Moreover, for $Y_a \geq m_a$, $d\left(Y_a\right) = \varphi_a\left(Y_a\right) - m_a$, which is nondecreasing in Y_a. Hence, there is a $y^* \in \left[m_a,\infty\right) \cup \left\{\infty\right\}$ such that $d\left(Y_a\right) > 0$ if and only if $Y_a > y^*$. Moreover, since the measure of agents is 2 and each is willing to pay at most ρ, Y_a is bounded by 2ρ. Let $\mu\left(y|u^a,\sigma\right)$ be the conditional density of Y_a at the realization y given the signals u^a and σ. Since the conditional (on u^a and σ) distribution of ζ^a is mutually absolutely continuous with respect to u^a, the conditional density has a well defined Radon-Nikodym derivative $\frac{\mu(y|u^a,\sigma)}{\mu(y|0,\sigma)}$ for each public signal σ. As noted by DeMarzo and Duffie [10, p. 88, n. 30], the measure μ can be chosen so that the Radon-Nikodym derivative $\frac{\mu(y|u^a,\sigma)}{\mu(y|0,\sigma)}$ is nondecreasing in y. Thus, for any

signal vector u^a,

$$
\begin{aligned}
\psi\left(u^{a}, \sigma\right) &= E\left(d\left(Y_{a}\right) | u^{a}, \sigma\right)=\int_{y=0}^{2\rho} d\left(y\right) \mu\left(y | u^{a}, \sigma\right) = \int_{y=0}^{2\rho} d\left(y\right) \frac{\mu\left(y | u^{a}, \sigma\right)}{\mu\left(y | 0, \sigma\right)} \mu\left(y | 0, \sigma\right) dy \\
&\geq \int_{y=0}^{2\rho} d\left(y\right) \frac{\mu\left(y^{*} | u^{a}, \sigma\right)}{\mu\left(y^{*} | 0, \sigma\right)} \mu\left(y | 0, \sigma\right) dy = \frac{\mu\left(y^{*} | u^{a}, \sigma\right)}{\mu\left(y^{*} | 0, \sigma\right)} \int_{y=0}^{2\rho} d\left(y\right) \mu\left(y | 0, \sigma\right) dy = 0
\end{aligned}
$$

Thus, $E\left[\varphi_{a}\left(Y_{a}\right) | u^{a}, \sigma\right]=E\left[\min \left\{m_{a}, Y_{a}\right\} | u^{a}, \sigma\right]+\psi\left(u^{a}, \sigma\right) \geq E\left[\min \left\{m_{a}, Y_{a}\right\} | u^{a}, \sigma\right]$.
Hence, by switching from the security $\varphi_{a}\left(Y_{a}\right)$ to the security $\min \left\{m_{a}, Y_{a}\right\}$, the bank weakly lowers $\widehat{\phi}_{a}\left(u^{a}, \sigma\right)$ (the expected payout of the security conditional on u^{a} and σ) while not changing $\widehat{\phi}_{a}\left(0, \sigma\right)$, thus weakly raising conditional profits $\pi\left(u^{a}, \sigma\right)$ and thus unconditional profits $E\left[\pi\left(u^{a}, \sigma\right)\right]$. This shows that the optimal security is debt. Q.E.D.

For the remainder, we need additional notation. Let $z_{\eta\ell}^{B}=r_{\eta\ell}^{B} p_{\eta\ell}^{B}$ denote the product of the interest rate charged to region B borrowers with credit score η in location ℓ and the proportion of these loans that are securitized. This quantity, which must lie between zero and $r_{\eta\ell}^{B}$, can be interpreted as the amount of loans that bank a securitizes, expressed in units of the face value $r_{\eta\ell}^{B}$ of these loans. Given $r_{\eta\ell}^{B}$, choosing $p_{\eta\ell}^{B}$ is clearly equivalent to choosing $z_{\eta\ell}^{B}$. With this change of variables,

$$
Y_{a}^{B}\left(\zeta^{B}\right)=\int_{\ell=0}^{1} \int_{\eta=0}^{1} x_{\eta\ell}^{B} z_{\eta\ell}^{B}\left[\eta S_{\ell}^{B} \int_{\nu=0}^{\underline{\nu}_{\eta\ell}^{B}} \nu dH\left(\nu | \eta, \ell\right)\right] dG\left(\eta, \ell\right). \tag{15}
$$

Bank a's Lagrangean equals its expected payoff Π_{a} plus constraint terms, which we write in a manner analogous to the integrals that appear in Π_{a}:

$$
\begin{aligned}
\mathcal{L} = {} & \Pi_{a}+\int_{\ell=0}^{1} \int_{\eta=0}^{1}\left(a_{\eta\ell} z_{\eta\ell}^{B}+b_{\eta\ell}\left(r_{\eta\ell}^{B}-z_{\eta\ell}^{B}\right)\right) \eta\left(1-H\left(\underline{\nu}_{\eta\ell}^{B} | \eta, \ell\right)\right) dG\left(\eta, \ell\right) \\
& +\int_{\ell=0}^{1} \int_{\eta=0}^{1}\left[c_{\eta\ell} r_{\eta\ell}^{A} \eta \int_{\nu=\bar{\nu}_{\eta\ell}^{A}}^{\bar{\nu}_{\eta\ell}} \nu dH\left(\nu | \eta, \ell\right)+d_{\eta\ell} r_{\eta\ell}^{A} \eta \int_{\nu=\underline{\nu}_{\eta\ell}^{A}}^{\bar{\nu}_{\eta\ell}^{A}} \nu dH\left(\nu | \eta, \ell\right)\right] dG\left(\eta, \ell\right)
\end{aligned}
$$

where $a_{\eta\ell}$, $b_{\eta\ell}$, $c_{\eta\ell}$, and $d_{\eta\ell}$ are Lagrange multipliers for the constraints $z_{\eta\ell}^{B} \geq 0$, $z_{\eta\ell}^{B} \leq r_{\eta\ell}^{B}$, $\bar{\nu}_{\eta\ell}^{A} \leq \bar{\nu}_{\eta\ell}$, and $\underline{\nu}_{\eta\ell}^{A} \leq \bar{\nu}_{\eta\ell}^{A}$, respectively. For technical reasons, we omit the constraint $r_{\eta\ell}^{B} \leq \rho$ and verify later that it holds. Bank b's Lagrangean, which is analogous, is omitted.

Proof of Claim 7: W.l.o.g. let $i = a$ and $j = b$. Since u^A and ζ^B are independent conditional on σ,

$$E\left[1\left(m_a > Y_a\right)\zeta_k^B\big|\, u^a, \sigma\right] = \int_{\zeta^B} \zeta_k^B\left[\int_{\zeta^A} 1\left(m_a > Y_a^B\left(\zeta^B\right) + Y_a^A\left(\zeta^A\right)\right) d\Gamma\left(\zeta^A|u^a, \sigma\right)\right] d\Gamma\left(\zeta^B|\sigma\right),$$

where $\Gamma\left(\zeta^B|\sigma\right)$ is the distribution function of ζ^B conditional on σ. By stochastic dominance, the interior integral is nonincreasing in u^a, so the double integral is as well. But stochastic dominance also implies $\widehat{\phi}_a\left(0, \sigma\right) \le \widehat{\phi}_a\left(u^a, \sigma\right)$. Hence $\Lambda_{ak}^{B0}\left(\sigma\right) \ge \Lambda_{ak}^B\left(\sigma\right)$ for all public signals σ, which proves that $\Omega_{a\ell}^B\left(\sigma\right) > 0$ since $\delta < 1$. Hence, $\Omega_{a\ell}^B = E\left(\Omega_{a\ell}^B\left(\sigma\right)\right) > 0$. Q.E.D.

Proof of Theorem 8: the proof consists of the following claims.

Claim 10 *If bank a competes for region B borrowers with credit score η in location ℓ, it bids a strictly positive interest rate $r_{\eta\ell}^B$.*

Claim 11 *If bank a competes for customers with credit score η in location ℓ in region B, then it includes all of them in its security: if $x_{\eta\ell}^B = 1$, then $p_{\eta\ell}^B = 1$.*

Claim 12 *Consider the group of agents with credit score η living in location ℓ in region B. Given the interest rate $r_{\eta\ell}^B$ offered by bank a, bank b responds as follows.*

1. *It lends to all agents whose private type ν exceeds*

$$\underline{\nu}_{\eta\ell}^B = \min\left\{\bar{\nu}_{\eta\ell}, \bar{\nu}_{\eta\ell}r_{\eta\ell}^{B*}/r_{\eta\ell}^B\right\}. \tag{16}$$

   In particular, it strictly prefers (not) to lend when a borrower's private type ν exceeds (respectively, is less than) $\bar{\nu}_{\eta\ell}r_{\eta\ell}^{B*}/r_{\eta\ell}^B$, and is indifferent when ν equals this expression.

2. *If bank b lends to some region B borrowers in this group (i.e., if $\underline{\nu}_{\eta\ell}^B < \bar{\nu}_{\eta\ell}$), then it securitizes all of these borrowers if $\Omega_{b\ell}^B > 0$ and none of them if $\Omega_{b\ell}^B < 0$.*

Claim 13 *If bank a competes for region B borrowers with credit score η in location ℓ (if $x_{\eta\ell}^B = 1$), it offers the interest rate $r_{\eta\ell}^B = \min\left\{\rho, r_{\eta\ell}^{B*}\right\}$ and lends to all borrowers in this group. If $r_{\eta\ell}^{B*} \leq \rho$, then bank b is just willing not to bid for the best borrower in this group: the borrower whose private type ν is $\bar{\nu}_{\eta\ell}$. If $r_{\eta\ell}^{B*} > \rho$, bank b strictly prefers not to bid for any borrowers in the group.*

Claim 14 *Bank a competes for region B borrowers with credit score η in location ℓ (i.e., it sets $x_{\eta\ell}^B = 1$) if and only if*

$$r_{\eta\ell}^B \left[\delta E\left(S_\ell^B\right) + \Omega_{a\ell}^B \right] \eta > 1. \tag{17}$$

This concludes the proof of Theorem 8. Q.E.D.

Proof of Claim 10: Suppose otherwise: $r_{\eta\ell}^B = 0$. Since $r_{\eta\ell}^B = 0$, $\partial X_a^B / \partial x_{\eta\ell}^B = 0$ and $\partial Y_a^B / \partial x_{\eta\ell}^B = 0$ (since $z_{\eta\ell}^B \leq r_{\eta\ell}^B$). Hence,

$$\frac{\partial \mathcal{L}}{\partial x_{\eta\ell}^B} = \delta E\left(\frac{\partial X_a^B}{\partial x_{\eta\ell}^B}\right) - \frac{\partial C_a^B}{\partial x_{\eta\ell}^B} + (1-\delta) E\left(\frac{\partial}{\partial x_{\eta\ell}^B} \frac{\widehat{\phi}_a\left(0, \sigma\right)^{\frac{1}{1-\delta}}}{\widehat{\phi}_a\left(u^a, \sigma\right)^{\frac{\delta}{1-\delta}}}\right) = -\frac{\partial C_a^B}{\partial x_{\eta\ell}^B} < 0.$$

Thus, $x_{\eta\ell}^B = 0$. Q.E.D.

Proof of Claim 11: The first order condition for $z_{\eta\ell}^B$ is

$$
\begin{aligned}
0 =\ & E\left(\frac{\widehat{\phi}_a\left(0, \sigma\right)^{\frac{1}{1-\delta}}}{\widehat{\phi}_a\left(u^a, \sigma\right)^{\frac{\delta}{1-\delta}}}\left(\frac{E\left(1\left(m_a > Y_a\right)\frac{\partial Y_a}{\partial z_{\eta\ell}^B}\middle| u^a = 0; \sigma\right)}{\widehat{\phi}_a\left(0, \sigma\right)} - \delta \frac{E\left(1\left(m_a > Y_a\right)\frac{\partial Y_a}{\partial z_{\eta\ell}^B}\middle| u^a, \sigma\right)}{\widehat{\phi}_a\left(u^a, \sigma\right)}\right)\right) \\
& + \left(a_{\eta\ell} - b_{\eta\ell}\right)\eta g\left(\eta, \ell\right)\int_{\nu=0}^{\nu_{\eta\ell}^B} \nu dH\left(\nu|\eta, \ell\right)
\end{aligned}
$$

However,

$$\frac{1}{\eta g\left(\eta, \ell\right)\int_{\nu=0}^{\nu_{\eta\ell}^B}\nu dH\left(\nu|\eta, \ell\right)}\frac{\partial Y_a}{\partial z_{\eta\ell}^B} = x_{\eta\ell}^B S_\ell^B = x_{\eta\ell}^B \sum_{k=1}^{K}\alpha_{k\ell}\zeta_k^B$$

Hence,

$$b_{\eta\ell} - a_{\eta\ell} = \begin{cases} 0 \text{ if } x_{\eta\ell}^B = 0 \\ \Omega_{a\ell}^B \text{ if } x_{\eta\ell}^B = 1 \end{cases} \tag{18}$$

By Claim 7, $\Omega^B_{a\ell} > 0$, whence $b_{\eta\ell} - a_{\eta\ell} > 0$ if $x^B_{\eta\ell} = 1$. But $a_{\eta\ell}$ and $b_{\eta\ell}$ are the Lagrange multipliers for the constraints $z^B_{\eta\ell} \geq 0$ and $z^B_{\eta\ell} \leq r^B_{\eta\ell}$, respectively. Thus, by Claim 10, either $a_{\eta\ell}$ or $b_{\eta\ell}$ must be zero. Together with (18), this implies that $b_{\eta\ell} = \Omega^B_{a\ell} > 0 = a_{\eta\ell}$, so $0 < z^B_{\eta\ell} = r^B_{\eta\ell}$. Q.E.D.

Proof of Claim 12: The derivatives of the bank's profits Π_b and the Lagrangean $\mathcal{L}$ with respect to $\bar{\nu}^B_{\eta\ell}$ are

$$\frac{\partial \Pi_b}{\partial \bar{\nu}^B_{\eta\ell}} = r^B_{\eta\ell} \eta \bar{\nu}^B_{\eta\ell} H' \left(\bar{\nu}^B_{\eta\ell} | \eta, \ell \right) g \left(\eta, \ell \right) \Omega^B_{b\ell} \tag{19}$$

and

$$\frac{\partial \mathcal{L}}{\partial \bar{\nu}^B_{\eta\ell}} = r^B_{\eta\ell} \eta \bar{\nu}^B_{\eta\ell} H' \left(\bar{\nu}^B_{\eta\ell} | \eta, \ell \right) g \left(\eta, \ell \right) \left[\Omega^B_{b\ell} + d_{\eta\ell} - c_{\eta\ell} \right]. \tag{20}$$

Since this must equal zero, it follows that

$$c_{\eta\ell} - d_{\eta\ell} = \Omega^B_{b\ell}. \tag{21}$$

The derivatives of the bank's profits Π_b and the Lagrangean L with respect to $\underline{\nu}^B_{\eta\ell}$ are

$$\frac{\partial \Pi_b}{\partial \underline{\nu}^B_{\eta\ell}} = \left(1 - \delta E \left(r^B_{\eta\ell} \eta \underline{\nu}^B_{\eta\ell} S^B_{\ell} \right) - r^B_{\eta\ell} \eta \underline{\nu}^B_{\eta\ell} \Omega^B_{b\ell} \right) H' \left(\underline{\nu}^B_{\eta\ell} | \eta, \ell \right) g \left(\eta, \ell \right) \tag{22}$$

and

$$\frac{\partial \mathcal{L}}{\partial \underline{\nu}^B_{\eta\ell}} = \frac{\partial \Pi_b}{\partial \underline{\nu}^B_{\eta\ell}} - d_{\eta\ell} r^B_{\eta\ell} \eta \underline{\nu}^B_{\eta\ell} H' \left(\underline{\nu}^B_{\eta\ell} | \eta, \ell \right) g \left(\eta, \ell \right). \tag{23}$$

First, suppose $\underline{\nu}^B_{\eta\ell} = \bar{\nu}_{\eta\ell}$. Then $\bar{\nu}^B_{\eta\ell} = \bar{\nu}_{\eta\ell}$ as well, so $\frac{\partial \Pi_b}{\partial \underline{\nu}^B_{\eta\ell}} \geq 0$ and $\frac{\partial \Pi_b}{\partial \underline{\nu}^B_{\eta\ell}} + \frac{\partial \Pi_b}{\partial \bar{\nu}^B_{\eta\ell}} \geq 0$. (The latter condition means that it is not optimal for the bank to lower both $\underline{\nu}^B_{\eta\ell}$ and $\bar{\nu}^B_{\eta\ell}$ while keeping them equal.) These two inequalities hold if and only if $1 - \delta E \left(r^B_{\eta\ell} \bar{\nu}_{\eta\ell} \eta S^B_{\ell} \right) - r^B_{\eta\ell} \bar{\nu}_{\eta\ell} \eta \left(\Omega^B_{b\ell} \right)^+ \geq 0$ which holds if and only if $r^{B*}_{\eta\ell} \geq r^B_{\eta\ell}$ by (10). This confirms that (16) holds when $\underline{\nu}^B_{\eta\ell} = \bar{\nu}_{\eta\ell}$.

Now suppose $\underline{\nu}^B_{\eta\ell} < \bar{\nu}_{\eta\ell}$. Recall that $c_{\eta\ell}$ and $d_{\eta\ell}$ are the Lagrange multipliers for the constraints $\bar{\nu}^B_{\eta\ell} \leq \bar{\nu}_{\eta\ell}$ and $\underline{\nu}^B_{\eta\ell} \leq \bar{\nu}^B_{\eta\ell}$, respectively. Only one of these can bind since $\underline{\nu}^B_{\eta\ell} < \bar{\nu}_{\eta\ell}$. Hence, either $c_{\eta\ell}$ or $d_{\eta\ell}$ is zero. Thus, by (21), $c_{\eta\ell} = \left(\Omega^B_{b\ell} \right)^+$ while $d_{\eta\ell} = \left(-\Omega^B_{b\ell} \right)^+$. Hence, bank b securitizes all of its borrowers in the group

$(c_{\eta\ell} > 0)$ if $\Omega_{b\ell}^B > 0$, and none of them $(d_{\eta\ell} > 0)$ if $\Omega_{b\ell}^B < 0$, as claimed. Moreover, by (21), (22), and (23), $0 = 1 - \delta E\left(r_{\eta\ell}^B \eta \underline{\nu}_{\eta\ell}^B S_\ell^B\right) - r_{\eta\ell}^B \eta \underline{\nu}_{\eta\ell}^B \left(\Omega_{b\ell}^B\right)^+$, which is equivalent to $\underline{\nu}_{\eta\ell}^B = \bar{\nu}_{\eta\ell} r_{\eta\ell}^{B*}/r_{\eta\ell}^B$. This shows that (16) holds when $\underline{\nu}_{\eta\ell}^B < \bar{\nu}_{\eta\ell}$ as well. Hence, (16) always holds. Finally, since only bank b knows ν, it strictly prefers (not) to lend to borrowers whose types ν exceed (respectively, are less than) $\bar{\nu}_{\eta\ell} r_{\eta\ell}^{B*}/r_{\eta\ell}^B$. Q.E.D.

Proof of Claim 13: The Lagrangean is not differentiable at the optimal interest rate $r_{\eta\ell}^B$. Hence, to find this optimal rate, we must consider the part of the Lagrangean in which $r_{\eta\ell}^B$ or $\underline{\nu}_{\eta\ell}^B$ (which depends on $r_{\eta\ell}^B$) appears. It is

$$\delta E\left(X_a^B\right) - C_a^B + (1-\delta) E\left(\frac{\widehat{\phi}_a\left(0,\sigma\right)^{\frac{1}{1-\delta}}}{\widehat{\phi}_a\left(u^a,\sigma\right)^{\frac{\delta}{1-\delta}}}\right)$$
$$+ \int_{\ell=0}^1 \int_{\eta=0}^1 \int_{\nu=0}^{\underline{\nu}_{\eta\ell}^B} \left(a_{\eta\ell} z_{\eta\ell}^B + b_{\eta\ell}\left(r_{\eta\ell}^B - z_{\eta\ell}^B\right)\right) \eta\nu dH\left(\nu|\eta,\ell\right) dG\left(\eta,\ell\right).$$

In addition, since the choice of $r_{\eta\ell}^B$ does not affect terms that involve credit scores $\eta' \neq \eta$ and locations $\ell' \neq \ell$, the optimal $r_{\eta\ell}^B$ is chosen to maximize

$$\delta E\left(S_\ell^B\right) \eta \int_{\nu=0}^{\underline{\nu}_{\eta\ell}^B} r_{\eta\ell}^B \nu dH\left(\nu|\eta,\ell\right) - \int_{\nu=0}^{\underline{\nu}_{\eta\ell}^B} dH\left(\nu|\eta,\ell\right)$$
$$+ (1-\delta) E\left(\frac{\widehat{\phi}_a\left(0,\sigma\right)^{\frac{1}{1-\delta}}}{\widehat{\phi}_a\left(u^a,\sigma\right)^{\frac{\delta}{1-\delta}}}\right) + \int_{\nu=0}^{\underline{\nu}_{\eta\ell}^B} \left(a_{\eta\ell} z_{\eta\ell}^B + b_{\eta\ell}\left(r_{\eta\ell}^B - z_{\eta\ell}^B\right)\right) \eta\nu dH\left(\nu|\eta,\ell\right).$$

Now, since $\frac{\partial Y_a}{\partial r_{\eta\ell}^B} = \frac{\partial}{\partial r_{\eta\ell}^B} \int_{\nu=0}^{\underline{\nu}_{\eta\ell}^B} x_{\eta\ell}^B z_{\eta\ell}^B \eta\nu dH\left(\nu|\eta,\ell\right) g\left(\eta,\ell\right) S_\ell^B$,

$$\frac{\partial}{\partial r_{\eta\ell}^B}\left((1-\delta) E\left(\frac{\widehat{\phi}_a\left(0,\sigma\right)^{\frac{1}{1-\delta}}}{\widehat{\phi}_a\left(u^a,\sigma\right)^{\frac{\delta}{1-\delta}}}\right)\right) = \frac{\partial}{\partial r_{\eta\ell}^B}\left(\int_{\nu=0}^{\underline{\nu}_{\eta\ell}^B} z_{\eta\ell}^B \eta\nu dH\left(\nu|\eta,\ell\right)\right) g\left(\eta,\ell\right) \Omega_{a\ell}^B.$$

By Claim 11, $a_{\eta\ell} = 0$ and $b_{\eta\ell} = \Omega_{a\ell}^B > 0$. Hence, $r_{\eta\ell}^B$ is chosen to maximize

$$c^{-1} \int_{\nu=0}^{\underline{\nu}_{\eta\ell}^B} \left(r_{\eta\ell}^B \nu - c\right) dH\left(\nu|\eta,\ell\right) \stackrel{d}{=} c^{-1} I\left(r_{\eta\ell}^B\right) \tag{24}$$

where $c^{-1} = \eta\left[\delta E\left(S_\ell^B\right) + \Omega_{a\ell}^B\right]$ is independent of $r_{\eta\ell}^B$. If $r_{\eta\ell}^B < r_{\eta\ell}^{B*}$, by (16), $\underline{\nu}_{\eta\ell}^B$ equals $\bar{\nu}_{\eta\ell}$ so small changes in $r_{\eta\ell}^B$ do not affect it. Hence, (24) is strictly increasing in $r_{\eta\ell}^B$, so the optimal $r_{\eta\ell}^B$ is at least $\min\left\{\rho, r_{\eta\ell}^{B*}\right\}$.

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If $r_{\eta\ell}^{B*} \geq \rho$, we are done. If $r_{\eta\ell}^{B*} < \rho$, it suffices to show that the optimal $r_{\eta\ell}^B$ is no greater than $r_{\eta\ell}^{B*}$. Let us write $r = r_{\eta\ell}^B$, $r^* = r_{\eta\ell}^{B*}$, and $H(\nu) = H(\nu|\eta, \ell)$ for brevity, and let $S \subset [0, \overline{\nu}_{\eta\ell}]$ be the support of ν for the given values of η and ℓ. Consider any $r > r^*$. We will show that if $I(r) > 0$, then $I'(r) < 0$. By (16), $\underline{\nu}_{\eta\ell}^B = \overline{\nu}_{\eta\ell} r^*/r < \overline{\nu}_{\eta\ell}$, so

$$I(r) = \int_{\nu=0}^{\overline{\nu}_{\eta\ell} r^*/r} (r\nu - c)\, dH(\nu) = \int_{\nu \in [0, \overline{\nu}_{\eta\ell} r^*/r] \cap S} (r\nu - c)\, dH(\nu) \qquad (25)$$

With the change of variables $x = r\nu$, $I(r) = \frac{1}{r} \int_{x \in [0, \overline{\nu}_{\eta\ell} r^*] \cap S'} (x - c) H'\left(\frac{x}{r}\right) dx$ where $S' = \{x \in [0, r] : x/r \in S\}$ is the support of x. Thus,

$$I'(r) = -\frac{1}{r^2}\left(\int_{x \in [0, \overline{\nu}_{\eta\ell} r^*] \cap S'} (x - c)\left[\frac{H'\left(\frac{x}{r}\right) + H''\left(\frac{x}{r}\right)\frac{x}{r}}{H'\left(\frac{x}{r}\right)}\right] H'\left(\frac{x}{r}\right) dx\right).$$

Changing variables back,

$$I'(r) = -\frac{1}{r}\left(\int_{\nu \in [0, \overline{\nu}_{\eta\ell} r^*/r] \cap S} (r\nu - c)\left[\frac{H'(\nu) + H''(\nu)\nu}{H'(\nu)}\right] dH(\nu)\right). \qquad (26)$$

For any functions $\varphi_0(\nu)$ and $\varphi_1(\nu)$, let $E^*(\varphi_0)$ and $Cov^*(\varphi_0, \varphi_1)$ denote the expectation of φ_0 and covariance of φ_0 and φ_1, both conditional on $\nu \in [0, \overline{\nu}_{\eta\ell} r^*/r] \cap S$. Then $I'(r) = -\frac{1}{r} E^*(xy) H(\overline{\nu}_{\eta\ell} r^*/r)$, where $x(\nu) = r\nu - c$ and $y(\nu) = \frac{H'(\nu) + H''(\nu)\nu}{H'(\nu)}$. By definition of covariance, $Cov^*(x, y) = E^*(xy) - E^*(x) E^*(y)$. Rearranging, $E^*(xy) = Cov^*(x, y) + E^*(x) E^*(y)$. Since $I(r) > 0$, $E^*(x) > 0$. By No Cream Skimming, $E^*(y) > 0$ and $Cov^*(x, y) \geq 0$. This proves that $E^*(xy) > 0$, so $I'(r) < 0$ as claimed.

Finally, by Lemma 12, $\underline{\nu}_{\eta\ell}^B = \min\{\overline{\nu}_{\eta\ell}, \overline{\nu}_{\eta\ell} r_{\eta\ell}^{B*}/r_{\eta\ell}^B\}$. Substituting for $r_{\eta\ell}^B$, $\underline{\nu}_{\eta\ell}^B = \min\{\overline{\nu}_{\eta\ell}, \overline{\nu}_{\eta\ell} r_{\eta\ell}^{B*}/\min\{\rho, r_{\eta\ell}^{B*}\}\} = \overline{\nu}_{\eta\ell}$: bank b does not lend to any borrowers in this group. Moreover, by Lemma 12, bank b strictly prefers (not) to lend to borrowers whose types ν exceed (respectively, are less than) $\overline{\nu}_{\eta\ell} r_{\eta\ell}^{B*}/r_{\eta\ell}^B$. If $r_{\eta\ell}^{B*} \leq \rho$, then $r_{\eta\ell}^B = \min\{\rho, r_{\eta\ell}^{B*}\} = r_{\eta\ell}^{B*}$: bank b is just willing not to bid for the best borrower in this group: the borrower whose private type ν is $\overline{\nu}_{\eta\ell}$. If $r_{\eta\ell}^{B*} > \rho$, then $r_{\eta\ell}^B = \rho < r_{\eta\ell}^{B*}$: bank b strictly prefers not to bid for any borrowers in the group. Q.E.D.

Proof of Claim 14: If the bank competes for these borrowers, then (a) by Claim 11 it securitizes every borrower who accepts (i.e., $z_{\eta\ell}^B = r_{\eta\ell}^B$) and (b) by Lemma 12, it outbids bank b for all borrowers in this group: $\underline{\nu}_{\eta\ell}^B = \bar{\nu}_{\eta\ell}$. By differentiating the Lagrangean $\mathcal{L}$ with respect to $x_{\eta\ell}^B$, one can easily verify that competing for these borrowers (setting $x_{\eta\ell}^B = 1$) raises bank a's profits if and only if (17) holds. Q.E.D.

References

[1] Agarwal, Sumit, and Robert Hauswald. September 2009. "The Choice Between Arms-Length and Inside Debt." Mimeo.

[2] Axelson, Ulf. 2007. "Security Design with Investor Private Information." Journal of Finance 62: 2587-2632.

[3] Biais, Bruno, and Thomas Mariotti. 2005. "Strategic Liquidity Supply and Security Design." Review of Economic Studies 72: 615-649.

[4] Boot, Arnoud W. A., and Anjan V. Thakor. 1993. "Security Design." Journal of Finance 48: 1349-1378.

[5] Broecker, Thorsten. 1990. "Credit-Worthiness Tests and Interbank Competition." Econometrica 58:429-452.

[6] Bubb, Ryan, and Alex Kaufman. 2009. "Securitization and Moral Hazard: Evidence from a Lender Cutoff Rule." Mimeo.

[7] Chemla, Gilles, and Christopher A. Hennessy. August 2011. "Skin in the Game and Moral Hazard." Mimeo.

[8] Degryse, Hans, and Steven Ongena. 2005. "Distance, Lending Relationships, and Competition." Journal of Finance 60:231-266.

[9] DeMarzo, Peter. 2005. "The Pooling and Tranching of Securities: A Model of Informed Intermediation." Review of Financial Studies 18:1-35.

[10] DeMarzo, Peter, and Darrell Duffie. 1999. "A Liquidity-Based Model of Security Design." Econometrica 67:65-99.

[11] Demange, Gabrielle and Guy Laroque. 1995. "Private Information and the Design of Securities." Journal of Economic Theory 65: 233-257.

[12] Demyanyk, Yuliya, and Otto Van Hemert. 2009. "Understanding the Subprime Mortgage Crisis." Review of Financial Studies 24:1848-1880.

[13] De Young, Robert, Dennis Glennon, and Peter Nigro. 2008. "Borrower–lender distance, credit scoring, and loan performance: Evidence from informational-opaque small business borrowers." Journal of Financial Intermediation 17:113-143.

[14] Elmer, Peter J. and Steven A. Seelig. 1998. "The Rising Long-Term Trend of Single-Family Mortgage Foreclosure Rates." Federal Deposit Insurance Corporation Working Paper #98-2.

[15] Elul, Ronel. 2011. "Securitization and Mortgage Default." Federal Reserve Bank of Cleveland, Working Paper No. 09-21/R.

[16] Gorton, Gary B., and J. G. Haubrich. 1990. "The Loan Sales Market." Research in Financial Services 2: 85–135.

[17] Hartman-Glaser, Barney, Tomasz Piskorski, and Alexei Tchistyi. 2010 "Optimal Securitization with Moral Hazard." Mimeo.

[18] Hauswald, Robert, and Robert Marquez. 2006. "Competition and Strategic Information Acquisition in Credit Markets." Review of Financial Studies 19:967-1000.

[19] Heuson, Andrea, Wayne Passmore, and Roger Sparks. 2001. "Credit Scoring and Mortgage Securitization: Implications for Mortgage Rates and Credit Availability." Journal of Real Estate Finance and Economics 23:337-363.

[20] Hill, Claire A. 1996. "Securitization: A Low-Cost Sweetener for Lemons." Washington University Law Quarterly 74:1061-1120.

[21] Jaffee, Dwight M., and Kenneth T. Rosen. 1990. "Mortgage Securitization Trends." Journal of Housing Research 1:117–137.

[22] Krainer, John, and Elizabeth Laderman. 2009. "Mortgage Loan Securitization and Relative Loan Performance." Federal Reserve Bank of San Francisco, Working Paper No. 2009-22.

[23] Loutskina, Elena, and Philip E. Strahan. 2011. "Informed and Uninformed Investment in Housing: The Downside of Diversification." Review of Financial Studies 24:1447-1480.

[24] Mian, Atif, and Amir Sufi. 2009. "The Consequences of Mortgage Credit Expansion: Evidence from the U.S. Mortgage Default Crisis." Quarterly Journal of Economics 124:1449-1496.

[25] Mistrulli, Paolo E., and Luca Casolaro. 2008. "Distance, Lending Technologies and Interest Rates." Mimeo, Bank of Italy.

[26] Myers, S., and N. Majluf. 1984. "Corporate Financing and Investment When Firms Have Information Shareholders Do Not Have." Journal of Financial Economics 13:187-221.

[27] Nachman, D. and T. Noe. 1994. "Optimal Design of Securities Under Asymmetric Information." Review of Financial Studies 7:1-44.

[28] Petersen, Mitchell A., and Raghuram G. Rajan. 2002. "Does Distance Still Matter? The Information Revolution in Small Business Lending." Journal of Finance 57:2533-2570.

[29] Rahi, Rohit. 1996. "Adverse Selection and Security Design." Review of Economic Studies 632: 287-300.

[30] Rajan, Uday, Amit Seru, and Vikrant Vig. 2010. "The Failure of Models That Predict Failure: Distance, Incentives and Defaults." Mimeo.

[31] Sharpe, Steven. 1990. "Asymmetric Information, Bank Lending and Implicit Contracts: A Stylized Model of Customer Relationships." Journal of Finance 55:1069-1087.

[32] Shin, Hyun S. 2009. "Securitization and Financial Stability." Economic Journal 119:309–332.

[33] Shleifer, Andrei, and Robert W. Vishny. 2010. "Unstable Banking." Journal of Financial Economics 97:306-318.